

ALGEBRA

Lesson 6: Using Substitution To Solve Algebraic Equations

1. EXPRESSIONS VS. EQUATIONS

Up to this point you have been working with algebraic expressions, which do not have an equal sign. To evaluate an expression, you have been given a numerical value for a variable, and then have been asked to calculate the value of the expression.

In this section, you will be working with algebraic equations, which do have an equal sign, and you will be using substitution to figure out the value of the variable.

This is an important distinction. Here it is again:

Expressions do not have an equal sign, and you **can't** figure out the value of the variable. Example: $x^2 + 5x - 12$ is an expression.

You **can't** figure out the value of x .

Equations do have an equal sign, and you **can** figure out the value of the variable. Example: $4x + 6 = 18$ is an equation.

You **can** figure out the value of x .

What Does It Mean To Solve and Algebraic Equation?

To solve an algebraic equation means to figure out the correct value for the variable in the equation.

To solve the equation $4x + 6 = 18$ means to figure out what value of x will give you a true equation, with equal quantities on both sides of the equal sign.

The solution for this equation is $x = 3$.

This means that when you plug 3 in for x , you will have a true statement with equal quantities on both sides of the equal sign.

$$4x + 6 = 18$$

$$4(3) + 6 = 18$$

$$12 + 6 = 18$$

$$18 = 18$$

This proves that 3 is the correct value for x in the equation $4x + 6 = 18$.

The solution to $4x + 6 = 18$ is $x = 3$.

In this lesson, you will be using substitution to solve equations. Later in the study of Algebra, you will be learning how to use algebra steps to solve equations.

2. EQUATIONS WITH VARIABLES ON ONE SIDE OF THE EQUAL SIGN

Example 1

What is the solution to $6x + 7 = 37$?

- a) $x = 2$ b) $x = 3$ c) $x = 4$ d) $x = 5$ e) $x = 6$

Remember, to find the solution means to figure out the value of x .

Since you are given five possible solutions, you can plug each one in and see if you get a true statement with equal quantities on both sides of the equal sign.

Try a) $x = 2$ $6x + 7 = 37$ Start by writing the original equation.
 $6(2) + 7 = 37$ Plug in 2 for x , then calculate.
 $12 + 7 = 37$
 $19 = 37$ 19 **does not** equal 37, so $x = 2$ **is not** the solution.

Notice that you are working down the left side of the equation. There is no work on the right side of the equation because there are no variables to plug in on the right side. The 37 on the right side of the equal sign gets carried down with each step until you have a single number on the left side of the equal sign to compare to the 37.

Try b) $x = 3$ $6x + 7 = 37$ Write the original equation.
 $6(3) + 7 = 37$ Plug in 3 for x , then calculate.
 $18 + 7 = 37$
 $25 = 37$ 25 **does not** equal 37, so $x = 3$ **is not** the solution.

Try c) $x = 4$ $6x + 7 = 37$ Write the original equation.
 $6(4) + 7 = 37$ Plug in 4 for x , then calculate.
 $24 + 7 = 37$
 $31 = 37$ 31 **does not** equal 37, so $x = 4$ **is not** the solution.

Try d) $x = 5$ $6x + 7 = 37$ Write the original equation.
 $6(5) + 7 = 37$ Plug in 5 for x , then calculate.
 $30 + 7 = 37$
 $37 = 37$ 37 **does** equal 37, so **d) $x = 5$ is the solution.**

This process is called trial and error.

Start with the original equation, then try a possible value for x by plugging it into the equation and calculating.

If you get a true equation with equal numbers on both sides of the equal sign, that is the correct value for x .

If you get an error, in other words, do not get equal quantities on both sides of the equal sign, the number you plugged in is not the correct value for x , so try another possible value for x .

Continue until you find the correct solution.

Example 2

If $x^2 + 3x - 3 = 85$, what is the value of x ?

- a) $x = 10$ b) $x = 9$ c) $x = 8$ d) $x = 7$ e) $x = 6$

In this equation, the variable x appears twice. A variable always has the same value throughout an equation, so plug in the same number every time x appears.

Try a) $x = 10$ $x^2 + 3x - 3 = 85$
 $10^2 + 3(10) - 3 = 85$
 $100 + 3(10) - 3 = 85$
 $100 + 30 - 3 = 85$
 $127 = 85$

Start by writing the original equation.
Plug in 10 for x in both places.
Calculate.

127 **does not** equal 85, so $x = 10$ is **not** the solution.

Try b) $x = 9$ $x^2 + 3x - 3 = 85$
 $9^2 + 3(9) - 3 = 85$
 $81 + 3(9) - 3 = 85$
 $81 + 27 - 3 = 85$
 $105 = 85$

105 **does not** equal 85, so $x = 9$ is **not** the solution.

Try c) $x = 8$ $x^2 + 3x - 3 = 85$
 $8^2 + 3(8) - 3 = 85$
 $64 + 3(8) - 3 = 85$
 $64 + 24 - 3 = 85$
 $85 = 85$

85 **does** equal 85, so **c) $x = 8$ is the solution.**

Example 3

If $4x + y = 22$, and $y = 10$, solve for x .

- a) $x = 6$ b) $x = 5$ c) $x = 3$ d) $x = 2$ e) $x = 1$

In this example, there are two variables, x and y . You are told $y = 10$, so 10 can replace y in the equation and stay there as you plug in possible values for x .

Try a) $x = 6$ $4x + y = 22$
 $4x + 10 = 22$
 $4(6) + 10 = 22$
 $24 + 10 = 22$
 $34 = 22$

Start by writing the original equation.
First, plug in the value for y that you are given.
Now, plug in the value for x that you are testing.

34 **does not** equal 22, so $x = 6$ is **not** the solution.

Try b) $x = 5$ $4x + 10 = 22$
 $4(5) + 10 = 22$
 $20 + 10 = 22$
 $30 = 22$

You know that $y = 10$, so keep it in place.

30 **does not** equal 22, so $x = 5$ is **not** the solution.

Try c) $x = 3$ $4x + 10 = 22$
 $4(3) + 10 = 22$
 $12 + 10 = 22$
 $22 = 22$

22 **does** equal 22, so **c) $x = 3$ is the solution.**

TIP – When you are using trial and error, cross off each multiple choice answer as you try it so you don't forget which ones you have already tried. If the problems and answers are on a computer screen, use scrap paper to note each answer as you try it.

Practice One Answers – p. 15

1. If $3x + 7 = 40$, what is the value of x ?
a) $x = 9$ b) $x = 10$ c) $x = 11$ d) $x = 12$ e) $x = 13$
2. What is the solution to $5x - 12 = 38$?
a) $x = 9$ b) $x = 10$ c) $x = 11$ d) $x = 12$ e) $x = 13$
3. If $8x + 37 = 61$, then $x =$?
a) $x = 5$ b) $x = 4$ c) $x = 3$ d) $x = 2$ e) $x = 1$
4. Find the value of x for the equation $7x - 23 = 26$.
a) $x = 3$ b) $x = 4$ c) $x = 6$ d) $x = 7$ e) $x = 8$
5. What is the solution to $4x^2 + 3x - 32 = 83$?
a) $x = 4$ b) $x = 5$ c) $x = 6$ d) $x = 7$ e) $x = 8$
6. What is the value of x in the equation $x^2 - 8x + 5 = -11$?
a) $x = 4$ b) $x = 5$ c) $x = 6$ d) $x = 7$ e) $x = 8$
7. If $5x + y = 68$, and $y = 8$, what is the value of x ?
a) $x = 5$ b) $x = 6$ c) $x = 8$ d) $x = 10$ e) $x = 12$
8. If $3y + 6x = 69$, and $y = 9$, then $x =$?
a) $x = 8$ b) $x = 7$ c) $x = 6$ d) $x = 5$ e) $x = 4$
9. If $3x^2 - 4y = 24$, and $y = 6$, find the value of x .
a) $x = 10$ b) $x = 6$ c) $x = 5$ d) $x = 4$ e) $x = 3$
10. If $y = 6$, and $y^2 + 2x^2 - 4y + 10x = 60$, what is the value of x ?
a) $x = 5$ b) $x = 4$ c) $x = 3$ d) $x = 6$ e) $x = 7$

3. COMPARISON: EXPRESSIONS VS. EQUATIONS

Let's continue with the comparison of expressions and equations that was discussed at the beginning of this lesson. There is difference between plugging in to calculate the value of an expression and plugging in to solve an equation. Compare the following two examples that illustrate this difference.

Example 1 Using substitution to **solve an equation**. This is like the problems you just did in Practice One.

What is the value of x if $17 + 6x = 35$?

- a) $x = 4$ b) $x = 3$ c) $x = 5$ d) $x = 8$ e) $x = 9$

In an equation problem like this, the question is: **What is the value of x ?** So, the multiple choice answers are possible answers to that question. **They are possible values for x , so they get plugged into the equation.** You do not know the value of x , so use trial and error until you find the value of x .

Try a) $x = 4$ $17 + 6x = 35$
 $17 + 6(4) = 35$
 $17 + 24 = 35$
 $41 = 35$
41 **does not** equal 35, so
 $x = 4$ **is not** correct.

Try b) $x = 3$ $17 + 6x = 35$
 $17 + 6(3) = 35$
 $17 + 18 = 35$
 $35 = 35$
35 **does** equal 35, so
b) $x = 3$ is correct.

Example 2 Using substitution to **evaluate an expression**. This is like the problems you did back in Lesson 5.

If $x = 2$, what is the value of $2x + 3 - x$?

- a) 8 b) 7 c) 5 d) 4 e) 3

In an expression problem like this, the question is: **What is the value of the whole expression?** So, the multiple choice answers are possible answers to that question. **They are not possible values for x , so they do not get plugged into the expression.**

You already know what x is. The problem tells you that $x = 2$, so that is the number you plug into the expression. You are not using trial and error here. You know the value of x , you plug it in, and calculate the value of the expression.

$2x + 3 - x$
 $2(2) + 3 - 2$
 $4 + 3 - 2$

5 **c) 5 is correct**

4. EQUATIONS WITH VARIABLES ON BOTH SIDES OF THE EQUAL SIGN

Example 1

What is the value of x in the equation $8x - 7 = 3x + 13$?

- a) $x = 5$ b) $x = 3$ c) $x = 4$ d) $x = 6$ e) $x = 7$

In this equation, the variable x appears on both sides of the equal sign. Plug in the value of x that you are testing in both places, and then simplify both sides of the equation.

Before, when x was only on one side of the equation, you did calculations only on one side of the equation. Now x is on both sides of the equation, so you do calculations on both sides of the equation. Work down each side, not across. Compare the results. If both sides are equal, the number you plugged in is the solution.

Try a) $x = 5$ $8x - 7 = 3x + 13$ Start by writing the original equation.
 $8(5) - 7 = 3(5) + 13$ Plug 5 in for x in both places.
 $40 - 7 = 15 + 13$ Calculate both sides of the equation.
 $33 = 28$ 33 **does not** equal 28, so $x = 5$ **is not** the solution.

Try b) $x = 3$ $8x - 7 = 3x + 13$ Write the original equation.
 $8(3) - 7 = 3(3) + 13$ Plug 3 in for x in both places.
 $24 - 7 = 9 + 13$ Calculate both sides of the equation.
 $17 = 22$ 17 **does not** equal 22, so $x = 3$ **is not** the solution.

Try c) $x = 4$ $8x - 7 = 3x + 13$ Write the original equation.
 $8(4) - 7 = 3(4) + 13$ Plug 4 in for x in both places.
 $32 - 7 = 12 + 13$ Calculate both sides of the equation.
 $25 = 25$ 25 **does** equal 25, so **c) $x = 4$ is the solution.**

Example 2

If $11x - 12 = 9x + 2$, then $x = ?$

- a) $x = 6$ b) $x = 7$ c) $x = 8$ d) $x = 9$ e) $x = 5$

Try a) $x = 6$ $11x - 12 = 9x + 2$ Start by writing the original equation.
 $11(6) - 12 = 9(6) + 2$
 $66 - 12 = 54 + 2$
 $54 = 56$ 54 **does not** equal 56, so $x = 6$ **is not** the solution.

Try b) $x = 7$ $11x - 12 = 9x + 2$
 $11(7) - 12 = 9(7) + 2$
 $77 - 12 = 63 + 2$
 $65 = 65$ 65 **does** equal 65, so **b) $x = 7$ is the solution.**

Practice Two Answers – p. 17

1. If $3x - 8 = x + 4$, what is the value of x ?
a) $x = 2$ b) $x = 4$ c) $x = 6$ d) $x = 8$ e) $x = 10$
2. What is the solution to the equation $5x + 3 = 8x - 3$?
a) $x = 1$ b) $x = 2$ c) $x = 3$ d) $x = 4$ e) $x = 5$
3. If $2x + 10 = 4x - 20$, what is the value of x ?
a) $x = 2$ b) $x = 10$ c) $x = 12$ d) $x = 18$ e) $x = 15$
4. Solve the equation $6x - 12 = 5x - 3$.
a) $x = 9$ b) $x = 8$ c) $x = 12$ d) $x = 7$ e) $x = 10$
5. What is the solution to $5(x + 15) = 4x^2$?
a) $x = 9$ b) $x = 7$ c) $x = 12$ d) $x = 5$ e) $x = 6$
6. If $3x^2 - 2 = 2(x + 19)$, find the value of x .
a) $x = 3$ b) $x = 4$ c) $x = 5$ d) $x = 6$ e) $x = 7$

5. ALGEBRAIC EQUATIONS WITH FRACTIONS AND SQUARE ROOTS

These examples may look intimidating because of the fractions and square roots they contain, but they work the same way as the other equations you have been solving with substitution. Plug in a value for x and use trial and error to solve.

Use your calculator to do operations with fractions and to calculate square roots.

**To enter a fraction on the calculator, enter the numerator, then the abc key, then the denominator. $\frac{1}{4}$ is entered as 1 abc 4.

**To reduce a fraction on the calculator, enter the fraction with the abc key, then press the equal key. For example, for $\frac{4}{8}$ enter 4 abc 8 = and you will get $\frac{1}{2}$.

**Calculate a square root by entering the number, then shift, then the x^2 key. $\sqrt{256}$ would be entered as 256 shift x^2 , and the answer 16 is displayed.

Example 1

If $\frac{1}{4}x = 10$, what is the value of x ?

- a) $x = 20$ b) $x = 36$ c) $x = 40$ d) $x = 55$ e) $x = 60$

Try a) $x = 20$ $\frac{1}{4}x = 10$ Start by writing the original equation.

$$\frac{1}{4}(20) = 10$$

Plug in 20 for x and multiply.

$$5 = 10$$

5 does not equal 10, so $x = 20$ **is not** the solution.

Try b) $x = 36$ $\frac{1}{4}x = 10$

$$\frac{1}{4}(36) = 10$$

$$9 = 10$$

9 does not equal 10, so $x = 36$ **is not** the solution.

Try c) $x = 40$ $\frac{1}{4}x = 10$

$$\frac{1}{4}(40) = 10$$

$$10 = 10$$

10 does equal 10, so **c) $x = 40$ is the solution.**

Example 2

If $\frac{3}{5}x + 15 = 30$, find the value of x .

- a) $x = 30$ b) $x = 25$ c) $x = 40$ d) $x = 55$ e) $x = 60$

Try a) $x = 30$ $\frac{3}{5}x + 15 = 30$ Start by writing the original equation.

$$\frac{3}{5}(30) + 15 = 30$$

$$18 + 15 = 30$$

$$33 = 30$$

33 does not equal 30, so $x = 30$ **is not** the solution.

Try b) $x = 25$ $\frac{3}{5}x + 15 = 30$

$$\frac{3}{5}(25) + 15 = 30$$

$$15 + 15 = 30$$

$$30 = 30$$

30 does equal 30, so **b) $x = 25$ is the solution.**

Example 3

If $\frac{x}{24} = \frac{5}{8}$, then $x = ?$

- a) $x = 8$ b) $x = 12$ c) $x = 15$ d) $x = 20$ e) $x = 30$

If you know how to solve a proportion, you can do that calculation and get your answer quickly without trial and error. That calculation is done by multiplying the diagonal numbers and dividing by the remaining number.

$$5 \times 24 \div 8 = x$$

$$15 = x$$

c) $x = 15$ is the answer.

OR, use trial and error to get a fraction equal to $\frac{5}{8}$.

Try a) $x = 8$

$$\frac{x}{24} = \frac{5}{8}$$

Start by writing the original equation, then plug in 8.

$$\frac{8}{24} = \frac{5}{8}$$

Reduce both fractions and see if they are equal.

$$\frac{1}{3} = \frac{5}{8}$$

$\frac{1}{3}$ **does not** equal $\frac{5}{8}$, so $x = 8$ **is not** the solution.

Try b) $x = 12$

$$\frac{x}{24} = \frac{5}{8}$$

Write the original equation and plug in 12.

$$\frac{12}{24} = \frac{5}{8}$$

Reduce both fractions and see if they are equal.

$$\frac{1}{2} = \frac{5}{8}$$

$\frac{1}{2}$ **does not** equal $\frac{5}{8}$, so $x = 12$ **is not** the solution.

Try c) $x = 15$

$$\frac{x}{24} = \frac{5}{8}$$

Write the original equation and plug in 15.

$$\frac{15}{24} = \frac{5}{8}$$

Reduce both fractions and see if they are equal.

$$\frac{5}{8} = \frac{5}{8}$$

$\frac{5}{8}$ **does** equal $\frac{5}{8}$, so **c) $x = 15$ is the solution.**

Example 4

What is the value of x when $\frac{x}{6} = \frac{2}{3}$?

- a) $x = 5$ b) $x = 4$ c) $x = 6$ d) $x = 8$ e) $x = 10$

Solve the proportion: $6 \times 2 \div 3 = 4$ **b) $x = 4$ is the answer.**

OR, use trial and error.

Try a) $x = 5$ $\frac{x}{6} = \frac{2}{3}$ Start by writing the original equation, then plug in 5.

$\frac{5}{6} = \frac{2}{3}$ Both fractions are already reduced and are not equal.

$\frac{5}{6} = \frac{2}{3}$ $\frac{5}{6}$ **does not** equal $\frac{2}{3}$, so $x = 5$ **is not** the answer.

Try b) $x = 4$ $\frac{x}{6} = \frac{2}{3}$ Write the original equation and plug in 4.

$\frac{4}{6} = \frac{2}{3}$ Reduce both fractions and see if they are equal

$\frac{2}{3} = \frac{2}{3}$ $\frac{2}{3}$ **does** equal $\frac{2}{3}$, so **b) $x = 4$ is the answer.**

Example 5

What is the solution to the following equation?

$$\sqrt{6x + 21} = 9$$

- a) $x = 6$ b) $x = 8$ c) $x = 10$ d) $x = 9$ e) $x = 12$

Try a) $x = 6$ $\sqrt{6x + 21} = 9$ Start by writing the original equation.

$$\sqrt{6(6) + 21} = 9$$
 Substitute.

$$\sqrt{36 + 21} = 9$$
 Calculate.

$$\sqrt{57} = 9$$

$$7.5 = 9$$
 7.5 does not equal 9, so $x = 6$ **is not** the solution.

Try b) $x = 8$ $\sqrt{6x + 21} = 9$

$$\sqrt{6(8) + 21} = 9$$

$$\sqrt{48 + 21} = 9$$

$$\sqrt{69} = 9$$

$$8.3 = 9$$
 8.3 does not equal 9, so $x = 8$ **is not** the solution.

Try c) $x = 10$ $\sqrt{6x + 21} = 9$

$$\sqrt{6(10) + 21} = 9$$

$$\sqrt{60 + 21} = 9$$

$$\sqrt{81} = 9$$

$$9 = 9$$
 9 does equal 9, so **c) $x = 10$ is the solution.**

Example 6

Determine the value of x in the following equation.

$$\sqrt{4x + 4} = 14$$

- a) $x = 18$ b) $x = 28$ c) $x = 48$ d) $x = 38$ e) $x = 52$

Try a) $x = 18$

$$\begin{aligned}\sqrt{4x + 4} &= 14 \\ \sqrt{4(18) + 4} &= 14 \\ \sqrt{72 + 4} &= 14 \\ \sqrt{76} &= 14 \\ 8.7 &= 14\end{aligned}$$

Start by writing the original equation.

Substitute.

Calculate.

8.7 **does not** equal 14, so $x = 18$ is **not** the solution.

Try b) $x = 28$

$$\begin{aligned}\sqrt{4x + 4} &= 14 \\ \sqrt{4(28) + 4} &= 14 \\ \sqrt{112 + 4} &= 14 \\ \sqrt{116} &= 14 \\ 10.8 &= 14\end{aligned}$$

10.8 **does not** equal 14, so $x = 28$ is **not** the solution.

Try c) $x = 48$

$$\begin{aligned}\sqrt{4x + 4} &= 14 \\ \sqrt{4(48) + 4} &= 14 \\ \sqrt{192 + 4} &= 14 \\ \sqrt{196} &= 14 \\ 14 &= 14\end{aligned}$$

14 **does** equal 14, so **c) $x = 48$ is the solution.**

Practice Three *Answers – p. 18*

1. If $\frac{2}{3}x = 24$, then $x = ?$

- a) $x = 16$ b) $x = 24$ c) $x = 36$ d) $x = 6$ e) $x = 4$

2. If $\frac{3}{5} = \frac{x}{25}$, determine the value of x .

- a) $x = 10$ b) $x = 15$ c) $x = 18$ d) $x = 20$ e) $x = 5$

3. What is the solution to the following equation?

$$\sqrt{4x + 5} = 3$$

- a) $x = 1$ b) $x = 2.2$ c) $x = 3$ d) $x = 4.5$ e) $x = 5$

4. What is the value of x in the equation $\frac{5}{8}x + 9 = 19$?

- a) $x = 10$ b) $x = 24$ c) $x = 20$ d) $x = 16$ e) $x = 4$

5. What is the value of x when $\frac{21}{x} = \frac{7}{8}$?
 a) $x = 31$ b) $x = 28$ c) $x = 18$ d) $x = 20$ e) $x = 24$
6. Solve for x in the equation $\sqrt{3x + 4} = 8$.
 a) $x = 15.3$ b) $x = 18$ c) $x = 20$ d) $x = 24$ e) $x = 30$
7. What is the solution to the equation $\frac{5}{9}x = 15$?
 a) $x = 27$ b) $x = 24$ c) $x = 28$ d) $x = 30$ e) $x = 36$
8. If $\frac{x}{16} = \frac{3}{4}$, what is the value of x ?
 a) $x = 8$ b) $x = 9$ c) $x = 10$ d) $x = 12$ e) $x = 15$
9. If $\sqrt{8x + 8} = 12$, then $x =$?
 a) $x = 15$ b) $x = 17$ c) $x = 19$ d) $x = 21$ e) $x = 23$

6. USING SUBSTITUTION IN ALGEBRAIC EQUATIONS TO CALCULATE AN AVERAGE

To calculate the average of a group of numbers, you add up all the numbers, and divide by the quantity of numbers you added up. You are probably used to average problems that give you a list of numbers and ask you to calculate an average.

What is the average of 20 and 30 ? Add $20 + 30$, then divide by 2.
 $(20 + 30) \div 2 = 25$. The average is 25.

What is the average of 5, 9, 14, and 20 ? Add $5 + 9 + 14 + 20$, then divide by 4.
 $(5 + 9 + 14 + 20) \div 4 = 12$. The average is 12.

Note that you must use parentheses around the values you are adding.

$(20 + 30) \div 2$ is correct. If you use $20 + 30 \div 2$ without parentheses, the order of operations says to do $30 \div 2$ first, and then add to 20. This would equal 35, which is not correct.

The problems in this section do not ask you to calculate an average. Instead, you are given an average, and all except one of the numbers that are added up to calculate the average. You are asked to find that missing number, which is the variable in the problem.

Example 1

If the average of 18 and x is 29, what is the value of x ?

- a) $x = 12$ b) $x = 48$ c) $x = 40$ d) $x = 23.5$ e) $x = 11$

You know that 18 plus some number, divided by 2 will give you 29. This problem can be expressed by the equation $(18 + x) \div 2 = 29$. Use trial and error to see which value of x produces an average of 29.

Try a) $x = 12$ $(18 + x) \div 2 = 29$ Set up an equation to get an average of 29.
 $(18 + 12) \div 2 = 29$ Plug 12 in for x , and calculate.
 $30 \div 2 = 29$
 $15 = 29$ **15 does not** equal 29, so $x = 12$ **is not** correct.

In this problem, you need an average of 29. Calculate down the left side of the equation, and at the end, compare the average you get when $x = 12$ with the average of 29 that you need.

Try b) $x = 48$ $(18 + x) \div 2 = 29$
 $(18 + 48) \div 2 = 29$ Plug 48 in for x , and calculate.
 $66 \div 2 = 29$
 $33 = 29$ **33 does not** equal 29, so $x = 48$ **is not** correct.

Try c) $x = 40$ $(18 + x) \div 2 = 29$
 $(18 + 40) \div 2 = 29$ Plug 40 in for x , and calculate.
 $58 \div 2 = 29$
 $29 = 29$ **29 does** equal 29, so **c) $x = 40$ is correct.**

CAREFUL – Notice that the question does not ask you to calculate an average. It gives you the average, and asks you to calculate a missing value. A very common mistake in problems like this is to get the average of the numbers stated in the question. In this problem, the mistake would be to get the average of 18 and 29.

Example 2

What is the value of x if the average of 6, 8, and x is equal to 10 ?

- a) $x = 13$ b) $x = 16$ c) $x = 8$ d) $x = 20$ e) $x = 24$

Notice that in this problem, three numbers are being averaged so you divide by 3.

Try a) $x = 13$ $(6 + 8 + x) \div 3 = 10$ Set up an equation to get an average of 10.
 $(6 + 8 + 13) \div 3 = 10$ Plug 13 in for x , and calculate.
 $27 \div 3 = 10$
 $9 = 10$ **9 does not** equal 10, so $x = 13$ **is not** correct.

Try b) $x = 16$ $(6 + 8 + x) \div 3 = 10$
 $(6 + 8 + 16) \div 3 = 10$ Plug 16 in for x , and calculate.
 $30 \div 3 = 10$
 $10 = 10$ **10 does** equal 10, so **b) $x = 16$ is correct.**

CAREFUL – Don't make the mistake of calculating the average of the numbers stated in the problem, 6, 8, and 10. You are not asked to calculate an average. You are given the average of three numbers, and asked to calculate the missing value.

Practice Four Answers – p. 20

1. If the average of 31 and x is 20, what is the value of x ?
a) $x = 8$ b) $x = 9$ c) $x = 10$ d) $x = 11$ e) $x = 12$
2. If the average of 6 and x is 12, find the value of x .
a) $x = 9$ b) $x = 20$ c) $x = 15$ d) $x = 18$ e) $x = 10$
3. What is the value of x if the average of x , 16, and 40 is 22 ?
a) $x = 39$ b) $x = 26$ c) $x = 10$ d) $x = 12$ e) $x = 15$
4. If the average of 3, 16, and x is 12, what is the value of x ?
a) $x = 15$ b) $x = 13$ c) $x = 27$ d) $x = 17$ e) $x = 14$
5. Find the value of x when the average of 4 and x is 7.
a) $x = 28$ b) $x = 11$ c) $x = 5$ d) $x = 7$ e) $x = 10$
6. Determine the value of x if the average of x and 20 is 30.
a) $x = 25$ b) $x = 40$ c) $x = 10$ d) $x = 30$ e) $x = 15$
7. If the average of 14, 16, 6, and x is 11, what is the value of x ?
a) $x = 10$ b) $x = 20$ c) $x = 3$ d) $x = 8$ e) $x = 15$

ANSWER KEY Lesson 6 Solving Algebraic Equations with Substitution

Practice One

1. If $3x + 7 = 40$, what is the value of x ? **Answer: $x = 11$**

- a) $x = 9$ b) $x = 10$ **c) $x = 11$** d) $x = 12$ e) $x = 13$

$$3x + 7 = 40$$

$$3(\mathbf{11}) + 7 = 40$$

$$33 + 7 = 40$$

$$40 = 40$$

2. What is the solution to $5x - 12 = 38$? **Answer: $x = 10$**

- a) $x = 9$ **b) $x = 10$** c) $x = 11$ d) $x = 12$ e) $x = 13$

$$5x - 12 = 38$$

$$5(\mathbf{10}) - 12 = 38$$

$$50 - 12 = 38$$

$$38 = 38$$

3. If $8x + 37 = 61$, then $x = ?$ **Answer: $x = 3$**

- a) $x = 5$ b) $x = 4$ **c) $x = 3$** d) $x = 2$ e) $x = 1$

$$8x + 37 = 61$$

$$8(\mathbf{3}) + 37 = 61$$

$$24 + 37 = 61$$

$$61 = 61$$

4. Find the value of x for the equation $7x - 23 = 26$. **Answer: $x = 7$**

- a) $x = 3$ b) $x = 4$ c) $x = 6$ **d) $x = 7$** e) $x = 8$

$$7x - 23 = 26$$

$$7(\mathbf{7}) - 23 = 26$$

$$49 - 23 = 26$$

$$26 = 26$$

5. What is the solution to $4x^2 + 3x - 32 = 83$? **Answer: $x = 5$**

- a) $x = 4$ **b) $x = 5$** c) $x = 6$ d) $x = 7$ e) $x = 8$

$$4x^2 + 3x - 32 = 83$$

$$4(\mathbf{5}^2) + 3(\mathbf{5}) - 32 = 83$$

$$4(25) + 3(\mathbf{5}) - 32 = 83$$

$$100 + 15 - 32 = 83$$

$$83 = 83$$

6. What is the value of x in the equation $x^2 - 8x + 5 = -11$? **Answer: $x = 4$**

- a) $x = 4$ b) $x = 5$ c) $x = 6$ d) $x = 7$ e) $x = 8$

$$x^2 - 8x + 5 = -11$$

$$4^2 - 8(4) + 5 = -11$$

$$16 - 8(4) + 5 = -11$$

$$16 - 32 + 5 = -11$$

$$-11 = -11$$

7. If $5x + y = 68$, and $y = 8$, what is the value of x ? **Answer: $x = 12$**

- a) $x = 5$ b) $x = 6$ c) $x = 8$ d) $x = 10$ e) $x = 12$

$$5x + y = 68$$

$$5x + 8 = 68$$

$$5(12) + 8 = 68$$

$$60 + 8 = 68$$

$$68 = 68$$

8. If $3y + 6x = 69$, and $y = 9$, then $x = ?$ **Answer: $x = 7$**

- a) $x = 8$ b) $x = 7$ c) $x = 6$ d) $x = 5$ e) $x = 4$

$$3y + 6x = 69$$

$$3(9) + 6x = 69$$

$$3(9) + 6(7) = 69$$

$$27 + 42 = 69$$

$$69 = 69$$

9. If $3x^2 - 4y = 24$, and $y = 6$, find the value of x . **Answer: $x = 4$**

- a) $x = 10$ b) $x = 6$ c) $x = 5$ d) $x = 4$ e) $x = 3$

$$3x^2 - 4y = 24$$

$$3x^2 - 4(6) = 24$$

$$3(4^2) - 4(6) = 24$$

$$3(16) - 4(6) = 24$$

$$48 - 24 = 24$$

$$24 = 24$$

10. If $y = 6$, and $y^2 + 2x^2 - 4y + 10x = 60$, what is the value of x ? **Answer: $x = 3$**

- a) $x = 5$ b) $x = 4$ c) $x = 3$ d) $x = 6$ e) $x = 7$

$$y^2 + 2x^2 - 4y + 10x = 60$$

$$6^2 + 2x^2 - 4(6) + 10x = 60$$

$$6^2 + (2)(3^2) - 4(6) + (10)(3) = 60$$

$$36 + (2)(9) - 4(6) + 10(3) = 60$$

$$36 + 18 - 24 + 30 = 60$$

$$60 = 60$$

Practice Two

1. If $3x - 8 = x + 4$, what is the value of x ? **Answer: $x = 6$**
a) $x = 2$ b) $x = 4$ **c) $x = 6$** d) $x = 8$ e) $x = 10$
 $3x - 8 = x + 4$
 $3(\mathbf{6}) - 8 = \mathbf{6} + 4$
 $18 - 8 = 10$
 $10 = 10$
2. What is the solution to the equation $5x + 3 = 8x - 3$? **Answer: $x = 2$**
a) $x = 1$ **b) $x = 2$** c) $x = 3$ d) $x = 4$ e) $x = 5$
 $5x + 3 = 8x - 3$
 $5(\mathbf{2}) + 3 = 8(\mathbf{2}) - 3$
 $10 + 3 = 16 - 3$
 $13 = 13$
3. If $2x + 10 = 4x - 20$, what is the value of x ? **Answer: $x = 15$**
a) $x = 2$ b) $x = 10$ c) $x = 12$ d) $x = 18$ **e) $x = 15$**
 $2x + 10 = 4x - 20$
 $2(\mathbf{15}) + 10 = 4(\mathbf{15}) - 20$
 $30 + 10 = 60 - 20$
 $40 = 40$
4. Solve the equation $6x - 12 = 5x - 3$. **Answer: $x = 9$**
a) **$x = 9$** b) $x = 8$ c) $x = 12$ d) $x = 7$ e) $x = 10$
 $6x - 12 = 5x - 3$
 $6(\mathbf{9}) - 12 = 5(\mathbf{9}) - 3$
 $54 - 12 = 45 - 3$
 $42 = 42$
5. What is the solution to $5(x + 15) = 4x^2$? **Answer: $x = 5$**
a) $x = 9$ b) $x = 7$ c) $x = 12$ **d) $x = 5$** e) $x = 6$
 $5(x + 15) = 4x^2$
 $5(\mathbf{5} + 15) = 4(\mathbf{5}^2)$
 $5(20) = 4(25)$
 $100 = 100$
6. If $3x^2 - 2 = 2(x + 19)$, find the value of x . **Answer: $x = 4$**
a) $x = 3$ **b) $x = 4$** c) $x = 5$ d) $x = 6$ e) $x = 7$
 $3x^2 - 2 = 2(x + 19)$
 $3(\mathbf{4}^2) - 2 = 2(\mathbf{4} + 19)$
 $3(16) - 2 = 2(23)$
 $48 - 2 = 46$
 $46 = 46$

Practice Three

1. If $\frac{2}{3}x = 24$, then $x = ?$ **Answer: $x = 36$**

- a) $x = 16$ b) $x = 24$ **c) $x = 36$** d) $x = 6$ e) $x = 4$

$$\frac{2}{3}x = 24$$

$$\left(\frac{2}{3}\right)(\mathbf{36}) = 24$$

$$24 = 24$$

2. If $\frac{3}{5} = \frac{x}{25}$, determine the value of x . **Answer: $x = 15$**

- a) $x = 10$ **b) $x = 15$** c) $x = 18$ d) $x = 20$ e) $x = 5$

$$\frac{3}{5} = \frac{x}{25}$$

$$\frac{3}{5} = \frac{\mathbf{15}}{25} \quad \text{reduce}$$

OR solve proportion: $3 \times 25 \div 5 = \mathbf{15}$

$$\frac{3}{5} = \frac{3}{5}$$

3. What is the solution to the following equation? **Answer: $x = 1$**

$$\sqrt{4x + 5} = 3$$

- a) **$x = 1$** b) $x = 2.2$ c) $x = 3$ d) $x = 4.5$ e) $x = 5$

$$\sqrt{4x + 5} = 3$$

$$\sqrt{4(\mathbf{1}) + 5} = 3$$

$$\sqrt{4 + 5} = 3$$

$$\sqrt{9} = 3$$

$$3 = 3$$

4. What is the value of x in the equation $\frac{5}{8}x + 9 = 19$? **Answer: $x = 16$**

- a) $x = 10$ b) $x = 24$ c) $x = 20$ **d) $x = 16$** e) $x = 4$

$$\frac{5}{8}x + 9 = 19$$

$$\frac{5}{8}(\mathbf{16}) + 9 = 19$$

$$10 + 9 = 19$$

$$19 = 19$$

5. What is the value of x when $\frac{21}{x} = \frac{7}{8}$? **Answer: $x = 24$**

- a) $x = 31$ b) $x = 28$ c) $x = 18$ d) $x = 20$ e) **$x = 24$**

$$\frac{21}{x} = \frac{7}{8}$$

$$\frac{21}{24} = \frac{7}{8} \quad \text{reduce}$$

$$\frac{7}{8} = \frac{7}{8}$$

OR solve proportion: $21 \times 8 \div 7 = 24$

6. Solve for x in the equation $\sqrt{3x + 4} = 8$. **Answer: $x = 20$**

- a) $x = 15.3$ b) $x = 18$ c) **$x = 20$** d) $x = 24$ e) $x = 30$

$$\sqrt{3x + 4} = 8$$

$$\sqrt{3(20) + 4} = 8$$

$$\sqrt{60 + 4} = 8$$

$$\sqrt{64} = 8$$

$$8 = 8$$

7. What is the solution to the equation $\frac{5}{9}x = 15$? **Answer: $x = 27$**

- a) **$x = 27$** b) $x = 24$ c) $x = 28$ d) $x = 30$ e) $x = 36$

$$\frac{5}{9}x = 15$$

$$\frac{5}{9}(27) = 15$$

$$15 = 15$$

8. If $\frac{x}{16} = \frac{3}{4}$, what is the value of x ? **Answer: $x = 12$**

- a) $x = 8$ b) $x = 9$ c) $x = 10$ d) **$x = 12$** e) $x = 15$

$$\frac{x}{16} = \frac{3}{4}$$

$$\frac{12}{16} = \frac{3}{4} \quad \text{reduce}$$

$$\frac{3}{4} = \frac{3}{4}$$

OR solve proportion: $3 \times 16 \div 4 = 12$

9. If $\sqrt{8x + 8} = 12$, then $x = ?$ **Answer: $x = 17$**

- a) $x = 15$ **b) $x = 17$** c) $x = 19$ d) $x = 21$ e) $x = 23$

$$\sqrt{8x + 8} = 12$$

$$\sqrt{8(\mathbf{17}) + 8} = 12$$

$$\sqrt{136 + 8} = 12$$

$$\sqrt{144} = 12$$

$$12 = 12$$

Practice Four

1. If the average of 31 and x is 20, what is the value of x ? **Answer: $x = 9$**

- a) $x = 8$ **b) $x = 9$** c) $x = 10$ d) $x = 11$ e) $x = 12$

$$(31 + x) \div 2 = 20$$

$$(31 + \mathbf{9}) \div 2 = 20$$

$$40 \div 2 = 20$$

$$20 = 20$$

2. If the average of 6 and x is 12, find the value of x . **Answer: $x = 18$**

- a) $x = 9$ b) $x = 20$ c) $x = 15$ **d) $x = 18$** e) $x = 10$

$$(6 + x) \div 2 = 12$$

$$(6 + \mathbf{18}) \div 2 = 12$$

$$24 \div 2 = 12$$

$$12 = 12$$

3. What is the value of x if the average of x , 16, and 40 is 22? **Answer: $x = 10$**

- a) $x = 39$ b) $x = 26$ **c) $x = 10$** d) $x = 12$ e) $x = 15$

$$(x + 16 + 40) \div 3 = 22$$

$$(\mathbf{10} + 16 + 40) \div 3 = 22$$

$$66 \div 3 = 22$$

$$22 = 22$$

4. If the average of 3, 16, and x is 12, what is the value of x ? **Answer: $x = 17$**

- a) $x = 15$ b) $x = 13$ c) $x = 27$ **d) $x = 17$** e) $x = 14$

$$(3 + 16 + x) \div 3 = 12$$

$$(3 + 16 + \mathbf{17}) \div 3 = 12$$

$$36 \div 3 = 12$$

$$12 = 12$$

5. Find the value of x when the average of 4 and x is 7. **Answer: $x = 10$**

- a) $x = 28$ b) $x = 11$ c) $x = 5$ d) $x = 7$ **e) $x = 10$**

$$(4 + x) \div 2 = 7$$

$$(4 + \mathbf{10}) \div 2 = 7$$

$$14 \div 2 = 7$$

$$7 = 7$$

6. Determine the value of x if the average of x and 20 is 30. **Answer: $x = 40$**

- a) $x = 25$ **b) $x = 40$** c) $x = 10$ d) $x = 30$ e) $x = 15$

$$(x + 20) \div 2 = 30$$

$$(\mathbf{40} + 20) \div 2 = 30$$

$$60 \div 2 = 30$$

$$30 = 30$$

7. If the average of 14, 16, 6, and x is 11, what is the value of x ? **Answer: $x = 8$**

- a) $x = 10$ b) $x = 20$ c) $x = 3$ **d) $x = 8$** e) $x = 15$

$$(14 + 16 + 6 + x) \div 4 = 11$$

$$(14 + 16 + 6 + \mathbf{8}) \div 4 = 11$$

$$44 \div 4 = 11$$

$$11 = 11$$