PROPORTION

Lesson 1 Ratio and Proportion Mechanics

1. RATIOS

A ratio is a comparison of two numbers, and shows the way that the two numbers are related to each other. For example, if there are 3 cans of soup and 8 cans of tuna in the pantry, the ratio of soup to tuna is 3 to 8. This can be written several different ways:

3 to 8, or 3:8, or as a fraction, $\frac{3}{8}$.

The order of the words and numbers matters.
The number that is first in the ratio, or on the top of the fraction, has to match the word that comes first in the phrase.

The ratio of soup to tuna is expressed as 3 to 8, 3:8, or $\frac{3}{8}$.

The ratio of tuna to soup is expressed as 8 to 3, 8:3, or $\frac{8}{3}$.

Reducing Ratios

Ratios should be reduced to lowest terms, so a ratio of $\frac{8}{12}$ would be reduced to $\frac{2}{3}$.

TIP – to reduce a fraction on the calculator, enter it as a fraction using the abc key, then press the equal sign.

To reduce $\frac{8}{12}$ enter 8 abc 12 = and you will get $\frac{2}{3}$.

NOTE – Ratios that reduce to a whole number should be expressed in fraction form, with the number 1 in the denominator (bottom of fraction). For example, the ratio $\frac{12}{4}$ will reduce to 3 on your calculator, but should be written as $\frac{3}{1}$.

Example 1: Sue walks 8 miles every 2 weeks.

1. The ratio of miles to weeks is: 8 to 2 which reduces to 4 to 1
   or 8:2 which reduces to 4:1
   or $\frac{8}{2}$ which reduces to $\frac{4}{1}$

2. The ratio of weeks to miles is: 2 to 8 which reduces to 1 to 4
   or 2:8 which reduces to 1:4
   or $\frac{2}{8}$ which reduces to $\frac{1}{4}$

Example 2: A class has 11 boys and 12 girls.

1. The ratio of boys to girls is: 11 to 12 or 11:12 or $\frac{11}{12}$
2. The ratio of girls to boys is: 12 to 11 or 12:11 or $\frac{12}{11}$
3. The ratio of girls to students is: 12 to 23 or 12:23 or $\frac{12}{23}$

NOTE – To form the last ratio, you have to add the number of boys plus the number of girls to get the number of students.

Reducing Improper Fraction Ratios

If you use the abc key to reduce an improper fraction ratio (top number is bigger than bottom number), you will get a mixed number, not a ratio in fraction form.

For example, if you reduce the ratio $\frac{25}{10}$ by entering 25 abc 10 =
you will get $2\frac{1}{2}$ which is a mixed number, not a ratio in fraction form.

So, if you want a ratio in fraction form, reduce an improper fraction ratio by hand by dividing the same number into the top and bottom of the fraction.

Reduce $\frac{25}{10}$ by dividing 5 into both the top and bottom numbers to get $\frac{5}{2}$.

$\frac{25 \div 5}{10 \div 5} \rightarrow \frac{5}{2}$

OR - Use this little trick: Flip the top and bottom number of an improper fraction ratio, reduce on the calculator, and then flip your answer.

For example, to reduce the ratio $\frac{25}{10}$, flip the fraction and reduce $\frac{10}{25}$ on the calculator, then flip your result.

Enter 10 abc 25 = and get $\frac{2}{5}$. Flip the result to get $\frac{5}{2}$.

Remember – do this only if the top number is bigger than the bottom number in a fraction ratio you need to reduce.

Example 3: Darla has 10 pennies, 6 nickels, and 4 dimes.

1. The ratio of pennies to nickels is: 10 to 6 or 10:6 or $\frac{10}{6}$

   which reduces to: 5 to 3 5:3 $\frac{5}{3}$

   (Reduce the improper fraction ratio $\frac{10}{6}$ by hand, or reduce $\frac{6}{10}$ on the calculator and flip the result.)

2. The ratio of coins to dimes is: 20 to 4 or 20:4 or $\frac{20}{4}$

   which reduces to: 5 to 1 5:1 $\frac{5}{1}$

3. The ratio of coins to nickels is: 20 to 6 or 20:6 or $\frac{20}{6}$

   which reduces to: 10 to 3 10:3 $\frac{10}{3}$

   (Reduce the improper fraction ratio $\frac{20}{6}$ by hand, or reduce $\frac{6}{20}$ on the calculator and flip the result.)

Practice One *Write your ratios in fraction form.* *Answers – p. 6*

1. A 24-piece box of candy has 6 chocolates, 5 caramels, 10 jellies, and 3 mints.

a) What is the ratio of chocolates to caramels?

b) What is the ratio of caramels to chocolates?

c) What is the ratio of jellies to mints?

d) What is the ratio of mints to total number of candies?

e) What is the ratio of total number of candies to chocolates?

f) What is the ratio of total number of candies to jellies?

2. Sammy played 12 games of chess and won 7 of them.

a) What is the ratio of games won to games played?

b) What is the ratio of games played to games won?

c) What is the ratio of games won to games lost?

d) What is the ratio of games lost to games to games played?

2. PROPORTIONS

A proportion shows two ratios that are equal to each other.

For example, $\frac{2}{5} = \frac{4}{10}$ is a proportion because both fractions are equal.

You can test that they are equal by reducing both fractions. You will get the same result for both.

$\frac{2}{5}$ reduces to $\frac{2}{5}$ and $\frac{4}{10}$ reduces to $\frac{2}{5}$

You can also test that they are equal by simplifying both fractions with division. Think of the fraction bar as a division sign.

$2 \div 5 = 0.4$ and $4 \div 10 = 0.4$

Another test to prove the fractions are equal is to multiply cross products. This means that if you multiply the diagonals, you will get the same product both times.

$2 \times 10 = 20$ and $5 \times 4 = 20$

To multiply diagonals means to multiply the top of one fraction in a proportion times the bottom of the other fraction.
The word "product" means the answer of a multiplication problem.

Meaning of a Proportion

If 5 ounces of candy cost \$3, then 15 ounces of candy cost \$9.

This relationship can be shown by the proportion: $\frac{5}{\$3} = \frac{15}{\$9}$

This is often read as "5 is to \$3 as 15 is to \$9" meaning that 5 and \$3 have the same relationship to each other as 15 and \$9 have to each other.

This means that for every 5 ounces of candy, you will have to pay \$3, no matter how many ounces there are. If you double the amount of candy, you double the amount of money.

In our proportion above, the second ratio has tripled the amount of candy and tripled the amount of money, so there is more candy and more money represented in the second ratio. Since both ratios have the same relationship of candy to money, the ratios are equal to each other, even though the amounts of candy and money represented by each ratio are different.

3. Cross Multiplication

Notice that a proportion has four parts. There are two tops, called numerators, and two bottoms, called denominators. In a proportion where three of the parts are known, cross multiplication can be used to solve for the missing part.

This is done by multiplying the two diagonal numbers and dividing by the remaining number. The answer will be the missing part.

Examples – *x* stands for the missing part, also called the unknown.

1. $\frac{x}{12} = \frac{3}{4}$ $x = 12 \times 3 \div 4 = \mathbf{9}$ **or** $x = 3 \times 12 \div 4 = \mathbf{9}$ $\frac{\mathbf{9}}{12} = \frac{3}{4}$

2. $\frac{4}{7} = \frac{x}{63}$ $x = 4 \times 63 \div 7 = \mathbf{36}$ **or** $x = 63 \times 4 \div 7 = \mathbf{36}$ $\frac{4}{7} = \frac{\mathbf{36}}{63}$

3. $\frac{15}{x} = \frac{75}{45}$ $x = 15 \times 45 \div 75 = \mathbf{9}$ **or** $x = 45 \times 15 \div 75 = \mathbf{9}$ $\frac{15}{\mathbf{9}} = \frac{75}{45}$

4. $\frac{24}{80} = \frac{3}{x}$ $x = 3 \times 80 \div 24 = \mathbf{10}$ **or** $x = 80 \times 3 \div 24 = \mathbf{10}$ $\frac{24}{80} = \frac{3}{\mathbf{10}}$

Test that the completed proportions are correct.

In example 1, the completed proportion is $\frac{9}{12} = \frac{3}{4}$.

Reduce both fractions. Both reduce to $\frac{3}{4}$.

Or, divide to simplify. $9 \div 12 = 0.75$ and $3 \div 4 = 0.75$

Or, multiply cross products. $12 \times 3 = 36$ and $9 \times 4 = 36$

Test the completed proportions in examples 2, 3, and 4 above.

Practice Two *Solve for x using cross multiplication.* *Answers – p. 6*

1. $\frac{x}{5} = \frac{24}{30}$ **2.** $\frac{9}{12} = \frac{3}{x}$

3. $\frac{5}{8} = \frac{x}{88}$ **4.** $\frac{4}{x} = \frac{36}{63}$

5. $\frac{10}{x} = \frac{25}{100}$ **6.** $\frac{x}{9} = \frac{75}{45}$

7. $\frac{9}{3} = \frac{12}{x}$ **8.** $\frac{24}{80} = \frac{x}{10}$

ANSWER KEY Lesson 1 Ratio and Proportion Mechanics

Practice One

1. A 24-piece box of candy has 6 chocolates, 5 caramels, 10 jellies, and 3 mints.

a) What is the ratio of chocolates to caramels? $\mathbf{\frac{6}{5}}$

b) What is the ratio of caramels to chocolates? $\mathbf{\frac{5}{6}}$

c) What is the ratio of jellies to mints? $\mathbf{\frac{10}{3}}$

d) What is the ratio of mints to total number of candies? $\frac{3}{24}$ reduces to $\mathbf{\frac{1}{8}}$

e) What is the ratio of total number of candies to chocolates? $\frac{24}{6}$ reduces to $\mathbf{\frac{4}{1}}$

f) What is the ratio of total number of candies to jellies? $\frac{24}{10}$ reduces to $\mathbf{\frac{12}{5}}$

(Reduce the improper fraction $\frac{24}{10}$ by hand, or reduce $\frac{10}{24}$ on the calculator and flip the result.)

2. Sammy played 12 games of chess and won 7 of them.

a) What is the ratio of games won to games played? $\mathbf{\frac{7}{12}}$

b) What is the ratio of games played to games won? $\mathbf{\frac{12}{7}}$

c) What is the ratio of games won to games lost? $\mathbf{\frac{7}{5}}$

(The problem does not provide the number of games lost. Subtract total games minus games won to get games lost. 12 – 7 = 5 games lost.)

d) What is the ratio of games lost to games to games played? $\mathbf{\frac{5}{12}}$

Practice Two

1. $\frac{x}{5} = \frac{24}{30}$ 5 x 24 ÷ 30 = $\boldsymbol{x = 4}$

2. $\frac{9}{12} = \frac{3}{x}$ 12 x 3 ÷ 9 = $\boldsymbol{x = 4}$

3. $\frac{5}{8} = \frac{x}{88}$ 5 x 88 ÷ 8 = $\boldsymbol{x = 55}$

4. $\frac{4}{x} = \frac{36}{63}$ 4 x 63 ÷ 36 = $\boldsymbol{x = 7}$

5. $\frac{10}{x} = \frac{25}{100}$ 10 x 100 ÷ 25 = $\boldsymbol{x = 40}$

6. $\frac{x}{9} = \frac{75}{45}$ 9 x 75 ÷ 45 = $\boldsymbol{x = 15}$

7. $\frac{9}{3} = \frac{12}{x}$ 3 x 12 ÷ 9 = $\boldsymbol{x = 4}$

8. $\frac{24}{80} = \frac{x}{10}$ 10 x 24 ÷ 80 = $\boldsymbol{x = 3}$