

ALGEBRA

Lesson 3 Order of Operations

1. ORDER OF OPERATIONS RULES

Operations refer to addition, subtraction, multiplication, and division. When you have an expression with more than one operation, you do not always perform them going from left to right. Rather, there is a specific order in which you must do them.

The order is:

1. **P**arentheses – Do all operations that are inside parentheses.
2. **E**xponents – Simplify all exponents.
3. **M**ultiply or **D**ivide, working from left to right.
4. **A**dd or **S**ubtract, working from left to right.

It may be helpful to use the phrase **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally to remember the order. **P E M D A S**.

2. BASIC ORDER OF OPERATIONS PROBLEMS

Example One

$6 + 18 - (9 - 6)$	First simplify the $9 - 6$ inside the parentheses to equal 3.
$6 + 18 - 3$	There are no exponents, multiplication, or division, so do the
$24 - 3$	addition and subtraction from left to right.
21	

Note that you should work down, not across. Make one change at a time, and rewrite the entire expression on the next line each time.

To do this, write the result of the calculation you have done, and carry down each number and operation sign that you have not changed. This seems like a lot of work, but it will keep things in order.

Example Two

$$4^2 - 5 + 2 \times 3$$

$$16 - 5 + 2 \times 3$$

$$16 - 5 + 6$$

$$11 + 6$$

$$17$$

There are no parentheses, so first simplify the exponent.

Next, do the multiplication. There is no division.

Next, do the addition and subtraction from left to right.

Note that the 4th step of the Order of Operations rules, *Add or Subtract, working from left to right*, means, working from left to right, do the operation if it is addition or if it is subtraction. It does **not** mean to do all the addition and then all the subtraction. You can see how this works in Example Two above where you first subtract $16 - 5$ to get 11, and next add 6, because working from left to right the subtraction comes first.

Compare Examples 3 and 4, and see how the parentheses make a difference.

Example Three

$$(18 - 8) \times 2$$

$$10 \times 2$$

$$20$$

Simplify the $18 - 8$ inside the parentheses first.

Example Four

$$18 - 8 \times 2$$

$$18 - 16$$

$$2$$

Do the multiplication first.

Example Five

$$40 \div 5 \times (3 + 4)$$

$$40 \div 5 \times 7$$

$$8 \times 7$$

$$56$$

Simplify parentheses.

Divide.

Multiply.

Note that the third step of the Order of Operations rules, *Multiply or Divide, working from left to right*, means to do the operation if it is multiplication or if it is division. It does **not** mean to do all the multiplication and then all the division. In Example Five, divide $40 \div 5$ to get 8, and then multiply $\times 7$, because working from left to right the division comes first.

Example Six

$(4 + 1) \times 3^2 + (-2)$	Simplify parentheses.
$5 \times 3^2 + (-2)$	Simplify exponent.
$5 \times 9 + (-2)$	Multiply.
$45 + (-2)$	Add.
43	

Note that when you are on the step to simplify parentheses, it means to simplify parentheses that have operations inside, like the $(4 + 1)$ in Example Six. Example Six has another set of parentheses, (-2) , but you can't simplify what is inside these parentheses because there is no operation. There is nothing to actually do inside the parentheses. They are just used to separate the -2 from the addition sign.

Example Seven

$5(3 + 19) - 4(9 + 3)$	Simplify parentheses.
$5 \times 22 - 4 \times 12$	Multiply.
$110 - 48$	Subtract.
62	

Note that a number in front of parentheses means multiplication.

$5(3 + 19)$ is the same as $5 \times (3 + 19)$.

$4(9 + 3)$ is the same as $4 \times (9 + 3)$

Example Eight

$2(2 + 3)^2 \div (8 - 3)$	Simplify parentheses. Do both in one step if you want.
$2 \times 5^2 \div 5$	Simplify exponent. Don't forget to bring it down from above.
$2 \times 25 \div 5$	Multiply and divide, working from left to right.
$50 \div 5$	
10	

In Example Eight, note that the exponent ² applies to the result you get after simplifying what is inside the parentheses. So, you end up with 5^2 .

Practice OneSimplify using the order of operations. *Answers – p. 7*

- | | |
|----------------------------------|--|
| 1) $6(5 + 2)$ | 2) $5 + 2 \times 9$ |
| 3) $18 \div (3 + 3)$ | 4) $18 \div 3 + 3$ |
| 5) $4 + 15 \div 3$ | 6) $6 - 4 + 3$ |
| 7) $(25 - 5) \times 2^2$ | 8) $25 - 5 \times 2^2$ |
| 9) $4^2 - 3 \times 5$ | 10) $8 + (-2) \times (-6)$ |
| 11) $50 \div 5 - 2 \times 3 + 8$ | 12) $6(2 + 9) - 7^2$ |
| 13) $30 - 5 \times 2 + (-4)$ | 14) $6 \times 8 \div (3 + 1)$ |
| 15) $20 - 2^2 \times 5$ | 16) $(4 \times 2)^2 - 20 + 6 \times 2$ |

3. MORE ORDER OF OPERATIONS PROBLEMS**Example One**

$$5\left(\frac{5}{8} - \frac{1}{4}\right) + 3 \times \frac{1}{6} \quad \text{Simplify parentheses.}$$

$$5 \times \frac{3}{8} + 3 \times \frac{1}{6} \quad \text{Multiply.}$$

$$1\frac{7}{8} + \frac{1}{2} \quad \text{Add.}$$

$$2\frac{3}{8}$$

Don't let the fractions intimidate you. Enter fractions on the calculator by using the abc key to separate the parts of fractions and mixed numbers.

Enter the fraction $\frac{5}{8}$ as 5 abc 8.

The calculator will display 5_8.

Enter the mixed number $1\frac{7}{8}$ as 1 abc 7 abc 8.

The calculator will display 1_7_8.

Then calculator uses the symbol _ to separate the top of the fraction from the bottom of the fraction, and also to separate the whole number from the fraction in a mixed number.

Example Two

$$16 - (20 - 3 \times 2^2) + 3 \times 7$$

$$16 - (20 - 3 \times 4) + 3 \times 7$$

$$16 - (20 - 12) + 3 \times 7$$

$$16 - 8 + 3 \times 7$$

$$16 - 8 + 21$$

$$8 + 21$$

$$29$$

In example one, there are several operations inside the parentheses. Treat the expression inside the parentheses like a separate problem and use the order of operations to simplify it down to one number. Then use that result as you simplify the rest of the expression.

Example Three

$$\frac{4 + 4 \times 5}{3^2 + 4 - 1}$$

$$\frac{4 + 20}{9 + 4 - 1}$$

$$\frac{24}{13 - 1}$$

$$\frac{24}{12}$$

$$2$$

In example two, you have a fraction with operations in both the numerator (top of the fraction) and the denominator (bottom of the fraction). Use the order of operations rules to simplify the numerator and the denominator as separate problems. You will end up with a fraction that has one number on top and one number on the bottom. This is your answer. You may need to divide or reduce to put the answer in standard form.

TIP – Use division to simplify the fraction $\frac{24}{12}$. $\frac{24}{12} \rightarrow 24 \div 12 = 2$
Or, to reduce any fraction, enter it as a fraction on the calculator and hit the = key. Enter 24 abc 12 = and you will get 2.

Practice Two Simplify using the order of operations. *Answers – p. 8*

1) $5 + (6 + 3 \times 2^2) \div 2$

2) $\frac{5}{8} + \frac{1}{2} \times 1\frac{7}{8}$

3) $\frac{3^2 + (10 - 5)}{18 - 2 \times 8}$

4) $\frac{20 - 5 \times 3}{2 + 4 \times 2}$

5) $\left(\frac{2}{3}\right)^2 + 8 \times \frac{1}{2}$

6) $12 - \frac{2}{5}\left(6 - \frac{5}{8}\right)$

7) $8\left(\frac{3}{4} + \frac{1}{8}\right) - 4$

8) $40 \div 2 - (3^2 + 1)$

9) $20 - (20 - 5 \times 2)$

10) $\frac{9 - 2 \times 4 + 3}{4 + 18 \div 3 + 2}$

4. ORDER OF OPERATIONS ON THE CALCULATOR

The Casio fx-260 calculator is programmed to follow the order of operations rules. This means that you can enter an expression exactly as written from left to right, and the calculator will do the operations in the correct order.

$20 - 5 \times 3$ Enter $20 - 5 \times 3 =$
You will get 5. The calculator has done the multiplication first.

$5(3^2 + 4)$ Enter $5 \times (3 \ x^2 + 4) =$
You will get 65. The calculator has first simplified the parentheses, then multiplied times 5.
Note that you have to enter the x multiplication key between the 5 and the parentheses, even though it is not written in the problem.

$5 + (6 + 3 \times 2^2) \div 2$ Enter $5 + (6 + 3 \times 2 \ x^2) \div 2 =$
You will get 14. The calculator has first simplified the parentheses, then divided by 2, then added 5.

It is very easy to make a mistake if you are entering a long expression with parentheses and exponents, so if you use the calculator for this kind of problem, you might want to do the problem twice and make sure you get the same answer both times.

Another common mistake is to forget to hit the equal sign key at the end of entering the expression.

If you want some practice on the calculator, do some of the examples and practice problems from this lesson with the calculator.

ANSWER KEY Lesson 3 Order of Operations

Practice One

1) $6(5 + 2)$
 6×7
42

2) $5 + 2 \times 9$
 $5 + 18$
23

3) $18 \div (3 + 3)$
 $18 \div 6$
3

4) $18 \div 3 + 3$
 $6 + 3$
9

5) $4 + 15 \div 3$
 $4 + 5$
9

6) $6 - 4 + 3$
 $2 + 3$
5

7) $(25 - 5) \times 2^2$
 20×2^2
 20×4
80

8) $25 - 5 \times 2^2$
 $25 - 5 \times 4$
 $25 - 20$
5

9) $4^2 - 3 \times 5$
 $16 - 3 \times 5$
 $16 - 15$
1

10) $8 + (-2) \times (-6)$
 $8 + 12$
20

11) $50 \div 5 - 2 \times 3 + 8$
 $10 - 2 \times 3 + 8$
 $10 - 6 + 8$
 $4 + 8$
12

12) $6(2 + 9) - 7^2$
 $6 \times 11 - 7^2$
 $6 \times 11 - 49$
 $66 - 49$
17

13) $30 - 5 \times 2 + (-4)$
 $30 - 10 + (-4)$
 $20 + (-4)$
16

14) $6 \times 8 \div (3 + 1)$
 $6 \times 8 \div 4$
 $48 \div 4$
12

15) $20 - 2^2 \times 5$
 $20 - 4 \times 5$
 $20 - 20$
0

16) $(4 \times 2)^2 - 20 + 6 \times 2$
 $8^2 - 20 + 6 \times 2$
 $64 - 20 + 6 \times 2$
 $64 - 20 + 12$
 $44 + 12$
56

Practice Two

$$\begin{aligned} 1) \quad & 5 + (6 + 3 \times 2^2) \div 2 \\ & 5 + (6 + 3 \times 4) \div 2 \\ & 5 + (6 + 12) \div 2 \\ & 5 + 18 \div 2 \\ & 5 + 9 \\ & \mathbf{14} \end{aligned}$$

$$\begin{aligned} 2) \quad & \frac{5}{8} + \frac{1}{2} \times 1\frac{7}{8} \\ & \frac{5}{8} + \frac{15}{16} \\ & \mathbf{1\frac{9}{16}} \end{aligned}$$

$$\begin{aligned} 3) \quad & \frac{3^2 + (10 - 5)}{18 - 2 \times 8} \\ & \frac{3^2 + 5}{18 - 16} \\ & \frac{9 + 5}{2} \\ & \frac{14}{2} \\ & \mathbf{7} \end{aligned}$$

$$\begin{aligned} 4) \quad & \frac{20 - 5 \times 3}{2 + 4 \times 2} \\ & \frac{20 - 15}{2 + 8} \\ & \frac{5}{10} \\ & \mathbf{1/2} \end{aligned}$$

$$\begin{aligned} 5) \quad & \left(\frac{2}{3}\right)^2 + 8 \times \frac{1}{2} \\ & \frac{4}{9} + 8 \times \frac{1}{2} \\ & \frac{4}{9} + 4 \\ & \mathbf{4\frac{4}{9}} \end{aligned}$$

$$\begin{aligned} 6) \quad & 12 - \frac{2}{5} \left(6 - \frac{5}{8}\right) \\ & 12 - \frac{2}{5} \left(5\frac{3}{8}\right) \\ & 12 - 2\frac{3}{20} \\ & \mathbf{9\frac{17}{20}} \end{aligned}$$

$$\begin{aligned} 7) \quad & 8\left(\frac{3}{4} + \frac{1}{8}\right) - 4 \\ & 8\left(\frac{7}{8}\right) - 4 \\ & 7 - 4 \\ & \mathbf{3} \end{aligned}$$

$$\begin{aligned} 8) \quad & 40 \div 2 - (3^2 + 1) \\ & 40 \div 2 - (9 + 1) \\ & 40 \div 2 - 10 \\ & 20 - 10 \\ & \mathbf{10} \end{aligned}$$

9) $20 - (20 - 5 \times 2)$
 $20 - (20 - 10)$
 $20 - 10$
10

10) $\frac{9 - 2 \times 4 + 3}{4 + 18 \div 3 + 2}$
 $\frac{9 - 8 + 3}{4 + 6 + 2}$
 $\frac{1 + 3}{10 + 2}$
 $\frac{4}{12}$
1/3