

**GEOMETRY: CIRCLES**

**Lesson 2 Multi-Step Circle Word Problems**

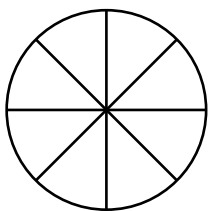
These are more complicated problems that may require multiple steps. Be sure to read each problem carefully. You may have to read a problem more than once to understand it.

Be sure to determine if a problem requires a circumference calculation or an area calculation, and then make sure you use the correct formula.

**Example 1**

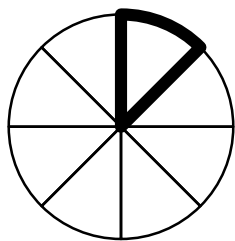
A circle is divided into 8 equal sections as shown below. If each divider line from the center of the circle to the edge of the circle is 9 inches long, what is the approximate distance around the outside edge of one of the sections?

- A. 37 in      B. 7 in      C. 57 in      D. 25      E. 72 in



First, decide if this is an area or a circumference problem. The problem asks for the distance around the outside edge of something, so this is a circumference/perimeter problem.

To get the distance around the edge of one of the sections means to get the length of the three edges of one section, shown by the darkened line below.



The problem tells you that each divider line is 9 inches from the center of circle to the edge, so you already know the lengths of the two straight edges of the section. To get the length of the third, curved edge, figure out the circumference of the whole circle, and then divide by 8 to get the length of the curved edge of one section.

**Step 1: Length of curved edge**

Figure out the circumference of the whole circle, and then divide by 8 to get the length of the curved edge of one section.

$$C = 2\pi r$$

Write out the formula.

$$C = 2(3.14)(9)$$

Substitute the values for pi and radius.

$$C = 56.52$$

Calculate.

$$56.52 \div 8 = 7.065 \text{ in}$$

Divide by 8 to get the length of the curved edge of one section.

**Step 2: Add together lengths of the 3 edges**

$$9 + 9 + 7 = 25 \text{ in}$$

**Answer: D. 25 in**

*Think about it* – How do you know that each divider line is a radius of the circle? The problem tells you that each divider line goes from the center of the circle to the edge of the circle.

**Example 2**

Using the information from Example 1, about how many square inches is of one of the sections?

A.  $32 \text{ in}^2$

B.  $254 \text{ in}^2$

C.  $81 \text{ in}^2$

D.  $72 \text{ in}^2$

E.  $23 \text{ in}^2$

First, decide if this is an area or a circumference problem. The problem asks for square inches, so this is an area problem.

To get the area of one section, calculate the area of the whole circle, and then divide by 8.

$$A = \pi r^2$$

Write out the formula.

$$A = (3.14)(9^2)$$

Substitute the values for pi and radius.

$$A = 3.14 \times 81$$

Calculate.

$$A = 254.34$$

Area of whole circle.

$$254.34 \div 8 = 31.7925$$

Divide by 8 to get the area of one section.

**Answer: A.  $32 \text{ in}^2$**

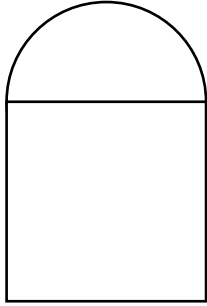
### **Example 3**

The shape below is comprised of a square and a half circle that are adjacent. If the length of one side of the square is 24 cm, what is the approximate area of the shape?

- A.  $576 \text{ cm}^2$       B.  $226 \text{ cm}^2$       C.  $452 \text{ cm}^2$       D.  $802 \text{ cm}^2$       E.  $864 \text{ cm}^2$

**NOTE** – Two shapes are **adjacent** when they are next to each other and share a side.

**NOTE** – A shape that is half of a circle can be called a **half circle** or a **semicircle**.



Get the areas of the square and the half circle that make up the shape, and add together.

#### **Step 1: Area of square**

$$24 \times 24 = 576 \text{ cm}^2$$

Area of a square is side x side (all sides are equal).

#### **Step 2: Area of half circle**

First get the area of the whole circle, then divide by 2.

Notice that the side of the square is the same as the diameter of the circle.

$$A = \pi r^2$$

Write out the formula.

$$A = (3.14)(12^2)$$

Substitute the values for pi and radius ( $\frac{1}{2}$  x diameter).

$$A = 3.14 \times 144$$

Calculate.

$$A = 452.16 \text{ cm}^2$$

Area of whole circle.

$$452.16 \div 2 = 226.08 \text{ cm}^2$$

Divide by 2 to get area of the half circle.

#### **Step 3: Area of whole shape**

$$576 + 226.08 = 802.08 \text{ cm}^2$$
 Add area of square + area of half circle.

**Answer: D.  $802 \text{ cm}^2$**

**Think about it** – How do you know that the side of the square is the same as the diameter of the full circle? It is the edge of the half circle, so it is the line that divides the full circle into two equal halves.

#### **Example 4**

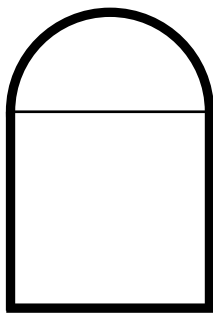
Using the diagram from Example 3, what is the distance around the outside edge of the shape?

- A. 134 cm      B. 110 cm      C. 75 cm      D. 96 cm      E. 171 cm

The problem asks for distance around the outside edge, so this is a perimeter/circumference problem.

To get the distance around the whole shape, you need the lengths of the three straight 24 cm sides of the square, plus the distance around the edge of the half circle, as shown by the darkened line below.

Note that you should not include all four of the square's sides because only three of them are part of the outside edge of the shape.



#### **Step 1: Circumference of half circle**

First get the circumference of a whole circle with a 24 cm diameter, and then divide by 2.

$$C = 2\pi r$$

Write out the formula.

$$C = 2(3.14)(12)$$

Substitute the values for pi and radius ( $\frac{1}{2}$  x diameter).

$$C = 75.36$$

Calculate to get circumference of the whole circle.

$$75.36 \div 2 = 37.68 \text{ cm}$$

Divide by 2 to get circumference of the half circle.

#### **Step 2: Add together circumference of half circle + 3 straight sides**

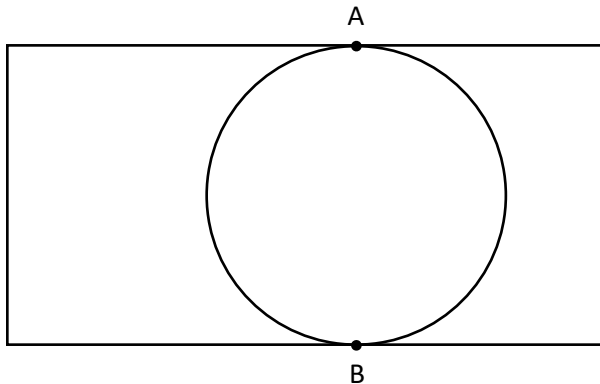
$$24 + 24 + 24 + 37.68 = 109.68$$

**Answer: B. 110 cm**

**Example 5**

A circular concrete area is built inside a rectangular grassy area as shown below, with the circle and the rectangle sharing only the two points shown, A and B. If the rectangle measures 12 feet x 24 feet, what is the approximate distance around the edge of the concrete circle?

- A. 12 ft      B. 24 ft      C. 32 ft      D. 75 ft      E. 38 ft



The problem asks for the distance around the edge of a circle, so this is a circumference problem. Think about what you need to calculate the circumference. The formula is  $C = 2\pi r$ , so if you can figure out the radius (or diameter) of the circle, you can use the formula to get circumference.

Start by labeling the diagram with the information you have. You can see that the line that forms a diameter between points A and B is the same length as the 12 ft side of the rectangle, as shown by the two darkened lines below. Now you know diameter = 12, so you can calculate circumference.

$C = 2\pi r$

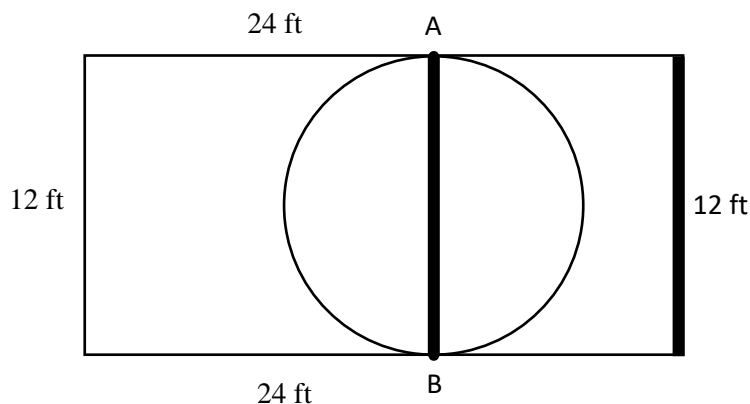
$C = 2(3.14)(6)$

$C = 37.68$

Write out the formula.

Substitute the values for pi and radius ( $\frac{1}{2}$  x diameter).

Calculate. **Answer: E. 38 ft**



**NOTE** – In the problem above, the two long sides of the rectangle are said to be **tangent** to the circle. **Tangent** means two shapes touch each other at just one point. The problem could have been worded: A circular concrete area is built inside a rectangular grassy area as shown below, where the long sides of the rectangle are **tangent** to the circle.

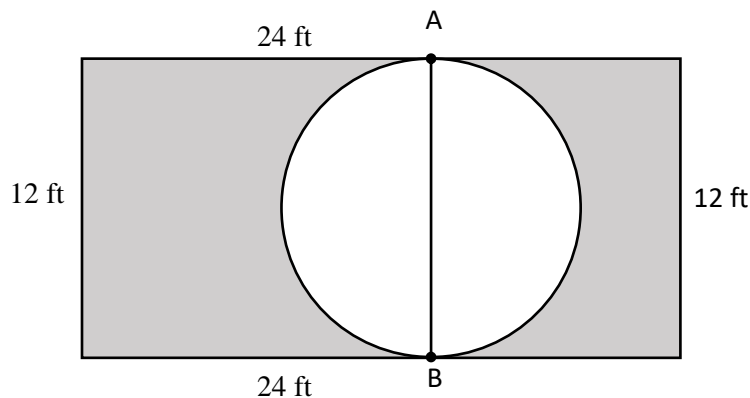
### **Example 6**

Using the diagram from Example 5, about how many square feet of grassy area remain after the concrete area is built?

- A.  $288 \text{ ft}^2$       B.  $113 \text{ ft}^2$       C.  $175 \text{ ft}^2$       D.  $144 \text{ ft}^2$       E.  $72 \text{ ft}^2$

The question asks for square feet, so you know this is an area problem.

The grassy area that remains is shown by the shaded area below.



We don't have a formula to calculate the area of the irregularly shaped shaded sections, but we do know how to get the area of the circle and the area of the rectangle. If you subtract the area of the circle from the area of the rectangle, you will be left with the area of the shaded sections.

#### **Step 1: Area of rectangle**

$$12 \times 24 = 288 \text{ ft}^2$$

Area = L x W This is for the entire rectangle, including the area covered by the concrete circle.

#### **Step 2: Area of circle**

$$A = \pi r^2$$

$$A = (3.14)(6^2)$$

$$A = 3.14 \times 36$$

$$A = 113.04 \text{ ft}^2$$

Write out the formula.

Substitute the values for pi and radius ( $\frac{1}{2}$  x diameter).

Calculate.

#### **Step 3: Area of shaded section**

$$288 - 113 = 175 \text{ ft}^2$$

Subtract the area of the circle from the area of the rectangle to get the area of the shaded sections.

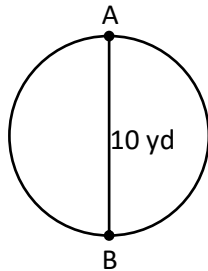
**Answer: C.  $175 \text{ ft}^2$**

**Practice – Multi-Step Circle Word Problems**

Answers – p. 10

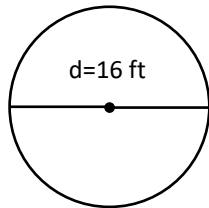
1. A line measuring 10 yard splits a circular game area into two equal halves, as shown below. Each player has to start at point A, run around the edge of the full circle back to point A, run across the line to point B, and then run around the edge of the full circle again, back to point B. About how many yards does each player have to run?

- A. 63 yd      B. 31 yd      C. 41 yd      D. 73 yd      E. 37 yd



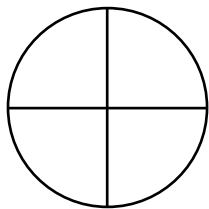
2. Which expression would you use to calculate the distance around the edge of the circle shown below?

- A.  $32\pi$       B.  $64\pi$       C.  $64\pi^2$       D.  $16\pi$       E.  $16\pi^2$



3. A circle is divided into 4 equal sections as shown below. If each dividing line is 7 inches from the center of the circle to the edge of the circle, what is the approximate area of one of the sections?

- A.  $154 \text{ in}^2$       B.  $83 \text{ in}^2$       C.  $38 \text{ in}^2$       D.  $49 \text{ in}^2$       E.  $28 \text{ in}^2$

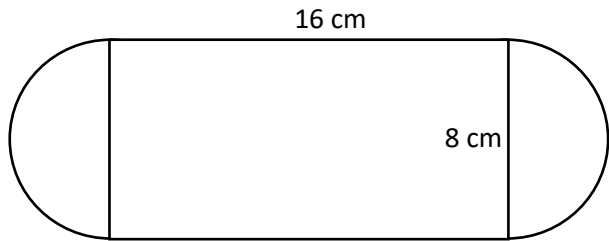


4. Using the diagram from problem #3, what is the approximate distance around the edge of one of the sections?

- A. 44 in      B. 25 in      C. 21 in      D. 11 in      E. 18 in

5. The shape below is comprised of a rectangle that measures 8 cm x 16 cm, and an adjacent half circle sharing the 8 cm line on each end of the rectangle. What is the approximate area of the shape in square cm?

- A.  $128 \text{ cm}^2$       B.  $384 \text{ cm}^2$       C.  $50 \text{ cm}^2$       D.  $178 \text{ cm}^2$       E.  $256 \text{ cm}^2$

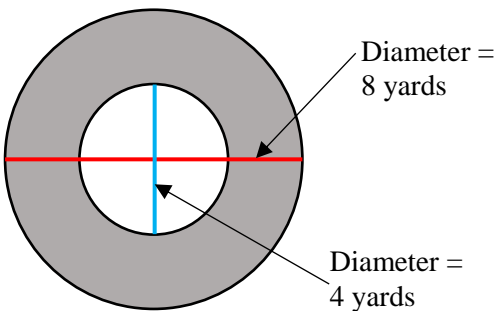


6. What is the approximate distance around the outside edge of the shape shown in problem #5?

- A. 73 cm      B. 48 cm      C. 128      D. 25 cm      E. 57 cm

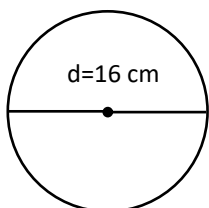
7. A white circle with a diameter of 4 yards is placed on top of a gray circle with a diameter of 8 yards. About how many square yards is the gray donut shaped area that remains visible after the white circle is placed on top of the gray circle?

- A.  $32 \text{ yd}^2$       B.  $38 \text{ yd}^2$       C.  $50 \text{ yd}^2$       D.  $13 \text{ yd}^2$       E.  $63 \text{ yd}^2$



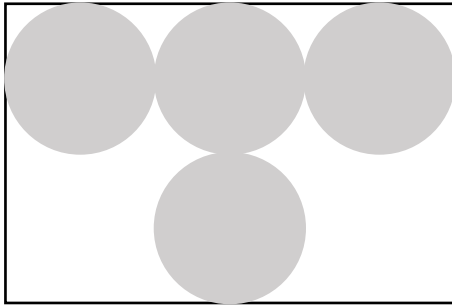
8. Which expression would you use to calculate the number of square cm in the circle shown below?

- A.  $64\pi$       B.  $256\pi$       C.  $64\pi^2$       D.  $16\pi$       E.  $32\pi^2$

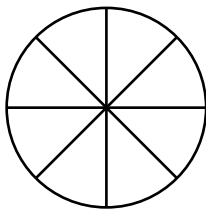




Questions 9 – 11 refer to the diagram below, with 4 equal circles inside a rectangle. The circles are tangent to the rectangle and to each other. (This means the circles share a single point of contact with each other and with the rectangle where they meet.) Each circle has a diameter of 6 feet.



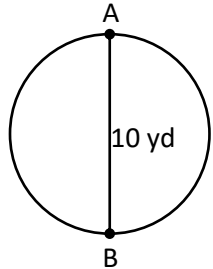
- 9.** What is the perimeter of the rectangle?  
 A. 216 ft      B. 30 ft      C. 60 ft      D. 24 ft      E. 36 ft
- 10.** What is the approximate area of the T shape formed by the 4 circles?  
 A. 28 ft<sup>2</sup>      B. 19 ft<sup>2</sup>      C. 75 ft<sup>2</sup>      D. 24 ft<sup>2</sup>      E. 113 ft<sup>2</sup>
- 11.** About how many square feet is the white section?  
 A. 216 ft<sup>2</sup>      B. 301 ft<sup>2</sup>      C. 24 ft<sup>2</sup>      D. 103 ft<sup>2</sup>      E. 329 ft<sup>2</sup>
- 12.** For an art project, a student is dividing a circle into 8 equal sections as shown below. The length of each divider line from the center of the circle to the edge is 13 inches. The student plans to cover each divider line and the outside edge of the circle with decorative trim that costs 15¢ per inch. How much will the trim cost?  
 A. \$27.90      B. \$15.60      C. \$104      D. \$2,790      E. \$81.64



## ANSWER KEY Lesson 2 Multi-Step Circle Word Problems

1. A line measuring 10 yard splits a circular game area into two equal halves, as shown below. Each player has to start at point A, run around the edge of the full circle back to point A, run across the line to point B, and then run around the edge of the full circle again, back to point B. About how many yards does each player have to run?

- A. 63 yd      B. 31 yd      C. 41 yd      **D. 73 yd**      E. 37 yd



Read the problem carefully to see that each player has to run around the edge of the circle twice (circumference), and across the divider line once.

### Step 1: Circumference

$$C = 2\pi r$$

$$C = 2(3.14)(5)$$

$$C = 31.4 \text{ yd}$$

Write out the formula.

Substitute the values for pi and radius ( $\frac{1}{2}$  x diameter).

Calculate.

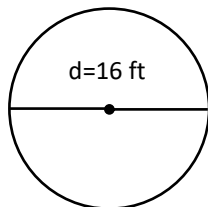
### Step 2: Total distance

$$31.4 + 31.4 + 10 = 72.8 \text{ yd}$$
 Twice around the circle plus once across the divider line.

**Answer: D. 73 yd**

2. Which expression would you use to calculate the distance around the edge of the circle shown below?

- A.  $32\pi$       B.  $64\pi$       C.  $64\pi^2$       **D.  $16\pi$**       E.  $16\pi^2$



The distance around the edge is circumference. Be sure to use the circumference formula and not the area formula.

Notice that you need an expression answer, and that all the answer choices include the  $\pi$  symbol.

$$C = 2\pi r$$

Write out the formula.

$$C = 2(\pi)(8)$$

Substitute the value for radius ( $\frac{1}{2}$  x diameter).

Don't substitute 3.14 for  $\pi$ , because  $\pi$  appears in all the answer choices.

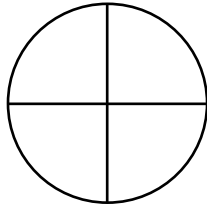
$$C = 16\pi$$

Calculate.

**Answer: D.  $16\pi$**

3. A circle is divided into 4 equal sections as shown below. If each dividing line is 7 inches from the center of the circle to the edge of the circle, what is the approximate area of one of the sections?

- A.  $154 \text{ in}^2$       B.  $83 \text{ in}^2$       C.  **$38 \text{ in}^2$**       D.  $49 \text{ in}^2$       E.  $28 \text{ in}^2$



Get the area of the whole circle, then divide by 4 to get the area of one section.

**Step 1: Area of circle**

$$A = \pi r^2$$

Write out the formula.

$$A = (3.14)(7^2)$$

Substitute the values for pi and the radius.

$$A = 3.14 \times 49$$

Calculate.

$$A = 153.86 \text{ in}^2$$

**Step 2: Area of one section**

$$153.86 \div 4 = 38.465 \text{ in}^2 \quad \text{Divide total area by 4 to get area of one section.}$$

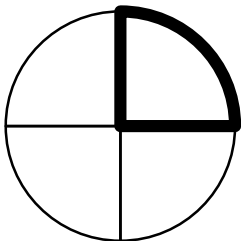
**Answer: C.  $38 \text{ in}^2$**

4. Using the diagram from problem #3, what is the approximate distance around the edge of one of the sections?

- A. 44 in      B. **25 in**      C. 21 in      D. 11 in      E. 18 in

The problem asks for the distance around the outside edge of something, so this is a circumference/perimeter problem.

To get the distance around the edge of one of the sections means to get the length of the three edges of one section, shown by the darkened line below.



The problem tells you that each divider line is 7 inches from the center of circle to the edge, so you already know the length of the two straight edges of the section.

To get the length of the third, curved edge, figure out the circumference of the whole circle, and then divide by 4 to get the length of the curved edge of one section.

**Step 1: Circumference of whole circle**

$$C = 2\pi r$$

Write out the formula.

$$C = 2(3.14)(7)$$

Substitute the values for pi and radius.

$$C = 43.96 \text{ in}$$

Calculate.

**Step 2: Curved edge of one section**

$$43.96 \div 4 = 10.99 \text{ in}$$

Whole circumference divided by 4 sections.

**Step 3: Total distance around one section**

$$7 + 7 + 11 = 25 \text{ in}$$

Add together the lengths of the 3 sides.

**Answer: B. 25 in**

5. The shape below is comprised of a rectangle that measures 8 cm x 16 cm, and an adjacent half circle sharing the 8 cm line on each end of the rectangle. What is the approximate area of the shape in square cm?

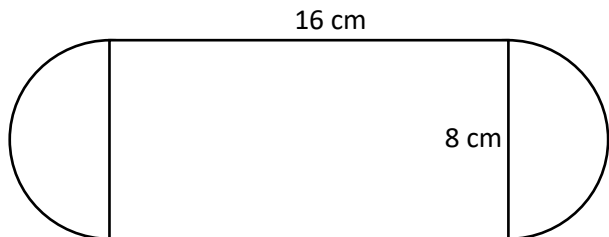
A.  $128 \text{ cm}^2$

B.  $384 \text{ cm}^2$

C.  $50 \text{ cm}^2$

**D.  $178 \text{ cm}^2$**

E.  $256 \text{ cm}^2$



This is a shape with 3 pieces. Get the area of each piece and then add together.

**Step 1: Area of rectangle**

$$8 \times 16 = 128 \text{ cm}^2$$

Area of rectangle = L x W

**Step 2: Area of half circles**

Each half circle has the same diameter, 8 cm. Since they have the same diameter, they are two halves of the same circle, or one full circle when combined.

$$A = \pi r^2$$

Write out the formula.

$$A = (3.14)(4^2)$$

Substitute the values for pi and radius ( $\frac{1}{2}$  x diameter).

$$A = 3.14 \times 16$$

Calculate.

$$A = 50.24 \text{ cm}^2$$

Area of whole circle is the same as area of 2 half circles.

**Step 3: Add together**

$$128 + 50 = 178 \text{ cm}^2$$

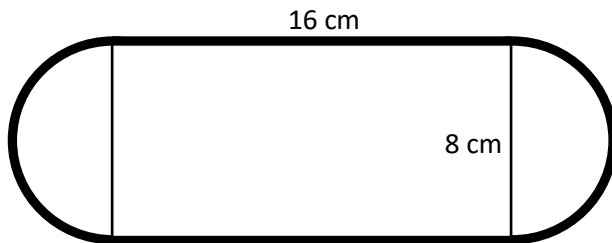
**Answer: D.  $178 \text{ cm}^2$**

6. What is the approximate distance around the outside edge of the shape shown in problem #5?

- A. 73 cm      B. 48 cm      C. 128      D. 25 cm      E. **57 cm**

The problem asks for distance around the outside edge, so this is a perimeter/circumference problem.

To get the distance around the whole shape, you need the lengths of the two straight 16 cm sides, plus the distance around the edges of the two half circles, as shown by the darkened line below. Note that you should not include the two 8 cm sides, because they are not part of the outside edge of the entire shape.



**Step 1: Circumference of half circles**

Each half circle has the same diameter, 8 cm. Since they have the same diameter, they are two halves of the same circle, or one full circle.

- $C = 2\pi r$       Write out the formula.  
 $C = 2(3.14)(4)$       Substitute the values for pi and radius ( $\frac{1}{2}$  x diameter).  
 $C = 25.12$  cm      Calculate. Circumference of whole circle is the same as circumference of the 2 half circles.

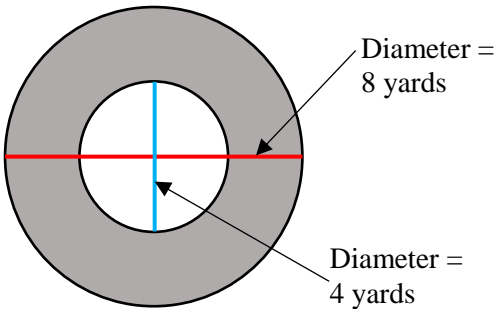
**Step 2: Add together circumference + 2 straight sides**

$$16 + 16 + 25.12 = 57.12 \text{ cm}$$

**Answer: E. 57 cm**

7. A white circle with a diameter of 4 yards is placed on top of a gray circle with a diameter of 8 yards. About how many square yards is the gray donut shaped area that remains visible after the white circle is placed on top of the gray circle?

- A. 32 yd<sup>2</sup>      B. 38 yd<sup>2</sup>      C. 50 yd<sup>2</sup>      D. 13 yd<sup>2</sup>      E. 63 yd<sup>2</sup>



The problem asks for square yards, so you know this is an area problem. Get the area of both circles. Subtract the small circle from the large circle to get the area of the gray donut shaped section.

**Step 1: Area of large gray circle**

- A =  $\pi r^2$       Write out the formula.
- A = (3.14)(4<sup>2</sup>)      Substitute the values for pi and radius (1/2 x diameter).
- A = 3.14 x 16      Calculate.
- A = 50.24 yd<sup>2</sup>      The area of 50.24 yd<sup>2</sup> is for the entire gray circle, including the part covered by the white circle.

**Step 2: Area of small white circle**

- A =  $\pi r^2$       Write out the formula.
- A = (3.14)(2<sup>2</sup>)      Substitute the values for pi and radius (1/2 x diameter).
- A = 3.14 x 4      Calculate.
- A = 12.56 yd<sup>2</sup>

**Step 3: Subtract**

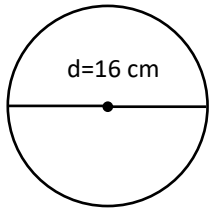
Subtract the small circle from the large circle to get the area of the visible gray donut shaped section.

$$50.24 - 12.56 = 37.68 \text{ yd}^2$$

**Answer: B. 38 yd<sup>2</sup>**

8. Which expression would you use to calculate the number of square cm in the circle shown below?

- A.  $64\pi$       B.  $256\pi$       C.  $64\pi^2$       D.  $16\pi$       E.  $32\pi^2$



The problem asks for square cm, so this is an area problem.  
Be sure to use the area formula and not the circumference formula.

Notice that you need an expression answer, and that all the answer choices include the  $\pi$  symbol.

$A = \pi r^2$       Write out the formula.

$A = \pi(8^2)$       Substitute the value for radius ( $\frac{1}{2}$  x diameter).

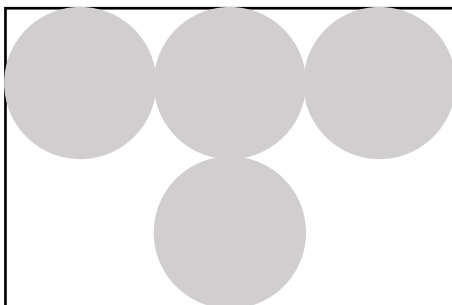
Don't substitute 3.14 for  $\pi$ , because  $\pi$  appears in all the answer choices.

$A = \pi(64)$       Calculate.

$A = 64\pi$       Write in standard form.

**Answer: A.  $64\pi$**

Questions 9 – 11 refer to the diagram below, with 4 equal circles inside a rectangle.  
The circles are tangent to the rectangle and to each other. (This means the circles share a single point of contact with each other and with the rectangle where they meet.)  
Each circle has a diameter of 6 feet.



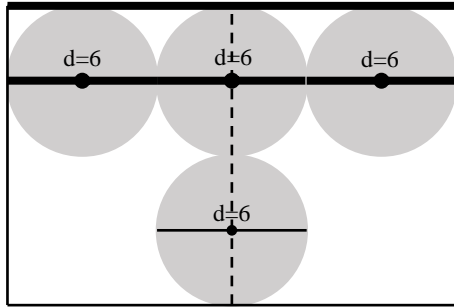
9. What is the perimeter of the rectangle?

- A. 216 ft      B. 30 ft      C. **60 ft**      D. 24 ft      E. 36 ft

What do you need to know in order to calculate the perimeter of the rectangle?

You need Length and Width.

If you don't see how to get them, fill in the diagram with the distances that you do know, and it will help you to find the distances you are looking for.



All four circles have a diameter of 6.

When combined, the 3 diameters lined up next to each other horizontally have the same length as the long side of the rectangle, as shown by the two heavy black lines.

$$\text{Length} = 3 \times 6 = 18 \text{ ft}$$

When combined, the 2 diameters lined up next to each other vertically have the same length as the short side of the rectangle, as shown by the two dashed lines.

$$\text{Width} = 2 \times 6 = 12 \text{ ft}$$

$$\text{Perimeter} = L + L + W + W = 18 + 18 + 12 + 12 = 60$$

**Answer: C. 60 ft**

10. What is the approximate area of the T shape formed by the 4 circles?

- A. 28 ft<sup>2</sup>      B. 19 ft<sup>2</sup>      C. 75 ft<sup>2</sup>      D. 24 ft<sup>2</sup>      E. **113 ft<sup>2</sup>**

All the circles have the same diameter, so all the areas will be the same. Get the area of one circle, and multiply x 4.

$$A = \pi r^2$$

$$A = (3.14)(3^2)$$

$$A = 3.14 \times 9$$

$$A = 28.26$$

$$28.26 \times 4 = 113.04$$

**Answer: E. 113 ft<sup>2</sup>**

Write out the formula.

Substitute the values for pi and radius ( $\frac{1}{2}$  x diameter).

Calculate.

Area of 1 circle.

Area of all 4 circles.



11. About how many square feet is the white section?

- A. 216 ft<sup>2</sup>      B. 301 ft<sup>2</sup>      C. 24 ft<sup>2</sup>      **D. 103 ft<sup>2</sup>**      E. 329 ft<sup>2</sup>

Subtract the area of the circles from the area of the whole rectangle, and you will be left with the area of the white section.

**Step 1: Area of rectangle**

$18 \times 12 = 216 \text{ ft}^2$       Multiply length x width, which you know from problem #9.

**Step 2: Area of gray circles**

$113 \text{ ft}^2$       From problem #10.

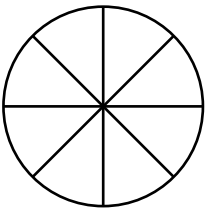
**Step 3: Subtract**

$216 - 113 = 103 \text{ ft}^2$       Subtract gray area from the area of the whole rectangle to get area of white section.

**Answer: D: 103 ft<sup>2</sup>**

12. For an art project, a student is dividing a circle into 8 equal sections as shown below. The length of each divider line from the center of the circle to the edge is 13 inches. The student plans to cover each divider line and the outside edge of the circle with decorative trim that costs 15¢ per inch. How much will the trim cost?

- A. **\$27.90**      B. \$15.60      C. \$104      D. \$2,790      E. \$81.64



To cover the outside edge of the circle and all the divider lines, you need the length of each divider line plus the circumference of the circle.

**Step 1: Circumference**

$C = 2\pi r$

$C = 2(3.14)(13)$

$C = 81.64 \text{ in}$

Write out the formula.

Substitute the values for pi and radius.

Calculate.

**Step 2: Length of divider lines**

$8 \times 13 = 104 \text{ in}$

Multiply 8 divider lines x 13 inches for each line.

**Step 3: Total length to be covered with trim**

$81.64 + 104 = 185.64$  in    Circumference + divider lines.

**Step 4: Cost of trim**

$186 \times \$0.15 = \$27.90$     Multiply inches x cost per inch.

**Answer: A. \$27.90**

**NOTE** – Round inches up before multiplying because you can't buy partial inches.

Use 0.15 for 15¢. Don't use 15, which means \$15.