PROBABILITY

Lesson 1 Beginning Probability

1. Basic Probability Calculation

What is Probability? It is a way of expressing how likely it is that something will happen.

Probability can be expressed as a fraction, decimal, or percent. We will start by using fractions, and work with decimals and percents later in the lesson.

Probability expressed as a fraction is calculated as: $\frac{\text{number of desired outcomes}}{\text{number of total possible outcomes}}$

Example One
There are 10 blue marbles and 15 red marbles in a bag. What is the probability that a marble picked at random will be red?

Total outcomes are the total number of marbles that could be picked.
Desired outcomes are the number of marbles that could be picked that meet the requirements of the problem, in this case, that are red.

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{15 red marbles}}{\text{25 total marbles}} \rightarrow \frac{15}{25} = \frac{3}{5}$

The probability that a randomly picked marble will be red is $\frac{3}{5}$.

Remember – To reduce a fraction on the calculator, enter with the abc key and hit the equal sign.
To reduce $\frac{15}{25}$ enter 15 abc key 25 = and $\frac{3}{5}$ is displayed.

Example Two
A 6-sided number cube (a die) numbered from 1 to 6 is rolled.
Possible rolls are: 1 2 3 4 5 6
Desired Outcomes are shown in **red** in the examples below.

A. What is the probability of rolling a 6?

How many of the possible rolls meet the requirement to be a 6?
Only 1 roll could result in a 6.
1 2 3 4 5 **6**

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{1 way to roll a 6}}{\text{6 possible rolls}} \rightarrow \frac{1}{6}$$

The probability of rolling a 6 is $\frac{1}{6}$.

B. What is the probability of rolling a 3?

How many of the possible rolls meet the requirement to be a 3?
Only 1 roll could result in a 3.
1 2 **3** 4 5 6

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{1 way to roll a 3}}{\text{6 possible rolls}} \rightarrow \frac{1}{6}$$

The probability of rolling a 3 is $\frac{1}{6}$.

C. What is the probability of rolling an even number?

How many of the possible rolls meet the requirement to be an even number?
There are 3 rolls that could result in an even number.
1 **2** 3 **4** 5 **6**

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{3 ways to roll an even number}}{\text{6 possible rolls}} \rightarrow \frac{3}{6} = \frac{1}{2}$$

The probability of rolling an even number is $\frac{1}{2}$.

D. What is the probability of rolling a 1 or a 5?

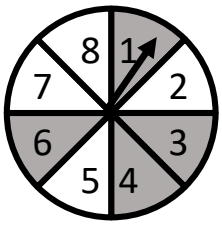
How many of the possible rolls meet the requirement to be a 1 or a 5?
There are 2 rolls that could result in a 1 or a 5.
1 2 3 4 **5** 6

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{2 ways to roll a 1 or a 5}}{\text{6 possible rolls}} \rightarrow \frac{2}{6} = \frac{1}{3}$$

The probability of rolling a 1 or a 5 is $\frac{1}{3}$.

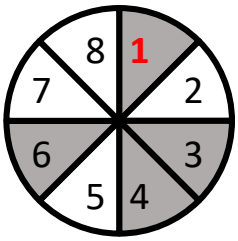
Example Three

A spinner is divided into 8 sections numbered 1 – 8, with 4 white sections and 4 gray sections, as shown.



A. What is the probability of spinning a 1?

How many of the sections meet the requirement to be a 1?

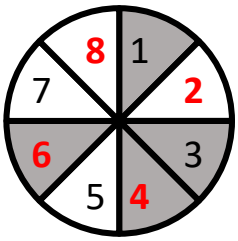


The section numbered 1 is the only section that meets the requirement.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{1 way to spin a 1}}{\text{8 possible spins}} \rightarrow \frac{1}{8}$$

B. What is the probability of spinning an even number?

How many of the sections meet the requirement to be an even number?

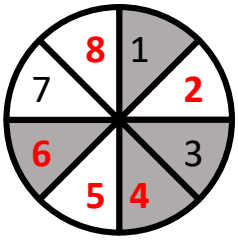


The sections numbered 2 4 6 8 meet the requirement.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{4 ways to spin an even number}}{\text{8 possible spins}} \rightarrow \frac{4}{8} = \frac{1}{2}$$

C. What is the probability of spinning a 5 or an even number?

How many of the sections meet the requirement to be 5 or even?
Go through each section one by one and ask: Is it 5 or is it even?
If it is either, count it as a desired outcome.
The use of the word "or" means a desired outcome must meet one of the requirements, but doesn't have to meet both.

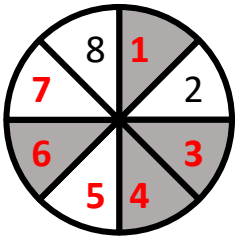


The sections numbered 2 4 5 6 8 meet one of the requirements.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{5 ways to spin a 5 or an even number}}{\text{8 possible spins}} \rightarrow \frac{5}{8}$$

D. What is the probability of spinning an odd or shaded section?

How many of the sections meet the requirement to be odd or shaded?
Go through each section one by one and ask: Is it odd or is it shaded?
If it is either, count it as a desired outcome.
The use of the word "or" means a desired outcome must meet one of the requirements, but doesn't have to meet both.



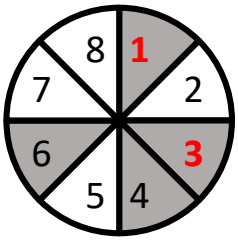
The sections numbered 1 3 4 5 6 7 meet one of the requirements.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{6 ways to spin odd or shaded}}{\text{8 possible spins}} \rightarrow \frac{6}{8} = \frac{3}{4}$$

CAREFUL – Do not count up all the odd sections and then all the shaded sections. That would give you 4 odd plus 4 shaded for 8 desired outcomes. This is incorrect because it double counts the sections that are both odd and shaded.

E. What is the probability of spinning an odd and shaded section?

How many of the sections meet the requirement to be odd and shaded?
Go through each section one by one and ask: Is it both odd and shaded?
If it meets both requirements, count it as a desired outcome.
The use of the word "and" means a desired outcome must meet both requirements.



The sections numbered 1 and 3 meet both requirements.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{2 ways to spin odd and shaded}}{\text{8 possible spins}} \rightarrow \frac{2}{8} = \frac{1}{4}$$

Notice the difference between questions D and E. In question D the section has to be ***either*** odd ***or*** shaded. In question E, the section has to be ***both*** odd ***and*** shaded.

Example Four

Below is an event center's calendar for the month of June.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

	Sports Event
	Music Event
	No Event

If a day is selected at random from the month of June, what is the probability of selecting:

A. A day with a sports event?
Count up all the days that are colored black.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{7 black days}}{\text{30 total days}} \rightarrow \frac{7}{30}$$

B. A day with no scheduled events?
Count up all the days that are colored white.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{17 white days}}{\text{30 total days}} \rightarrow \frac{17}{30}$$

CAREFUL – Count only the white squares that are actual days with numbers. Don't count the empty white squares.

C. A day with a sports or music event?
Count up all the days that are colored either black or gray.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{13 black or gray days}}{\text{30 total days}} \rightarrow \frac{13}{30}$$

D. A day that is even numbered and has a sporting event?

Count up all the days that are both even and black.
Go through each day one by one and ask: Is it even ***and*** is it colored black for a sporting event?
If it meets both requirements, count it as a desired outcome.
The use of the word "and" means a desired outcome must meet both requirements.

Days 2, 6, 14, and 28 meet both requirements.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{4 days that are black and even}}{\text{30 total days}} \rightarrow \frac{4}{30} = \frac{2}{15}$$

E. A day that is even numbered or has a sporting event?

Count up all the days that are either even or colored black for a sporting event.
Go through each day one by one and ask: Is it even ***or*** is it colored black for a sporting event?
If it meets either requirement, count it as a desired outcome.
The use of the word "or" means a desired outcome must meet one of the requirements, but doesn't have to meet both.

Days 2, 4, 6, 8, 10, 12, 13, 14, 16, 18, 20, 22, 23, 24, 26, 27, 28, and 30 meet one of the requirements.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{18 days that are black or even}}{\text{30 total days}} \rightarrow \frac{18}{30} = \frac{3}{5}$$

CAREFUL – Do not count up all the even days and then all the black days. That would give you 15 even plus 7 black for 22 desired outcomes. This is incorrect because it double counts the days that are both even and black.

Notice the difference between questions D and E. In question D the day has to be ***both*** even ***and*** a sporting event day. In question E, the day has to be ***either*** even ***or*** a sporting event day.

F. If it is Saturday, what is the probability of randomly selecting a day with no events?

The question limits the days to Saturdays, so total days are 4, not 30.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{1 white Saturday}}{\text{4 total Saturdays}} \rightarrow \frac{1}{4}$$

The day is not selected at random from the whole month. It is selected at random from all the Saturdays.

G. If it is Monday or Tuesday, what is the probability of randomly selecting a day with a music event?

The question limits the days to Mondays or Tuesdays, so total days are 10.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{3 gray Mondays or Tuesdays}}{\text{10 total Mondays or Tuesdays}} \rightarrow \frac{3}{10}$$

Practice One *Answers – p. 16*

1. A book club has 20 women and 5 men as members. If one person is selected at random to choose the book for next month, what is the probability that it will be a man?

A. $\frac{1}{4}$ B. $\frac{4}{1}$ C. $\frac{5}{1}$ D. $\frac{1}{5}$ E. $\frac{4}{5}$

Questions 2 – 4 refer to a bag of marbles that contains 20 red marbles, 30 blue marbles, 14 white marbles, and 40 black marbles.

2. What is the probability that a randomly selected marble will be black?

A. $\frac{13}{5}$ B. $\frac{5}{13}$ C. $\frac{40}{1}$ D. $\frac{4}{9}$ E. $\frac{4}{5}$

3. What is the probability that a randomly selected marble will be white?

A. $\frac{7}{52}$ B. $\frac{7}{30}$ C. $\frac{7}{90}$ D. $\frac{14}{50}$ E. $\frac{14}{90}$

4. What is the probability that a randomly selected marble will be either red or blue?

A. $\frac{15}{26}$ B. $\frac{5}{26}$ C. $\frac{15}{52}$ D. $\frac{24}{52}$ E. $\frac{25}{52}$

Questions 5 – 7 refer to the following table which shows the weights of a group of patients.

Weight in Pounds	Number of People
Under 100	15
100 – 125	25
126 – 150	40
151 – 175	35
176 – 200	25
Over 200	20

5. If a person is randomly selected what is the probability that the person will weigh 100 to 125 pounds?

A. $\frac{5}{28}$ B. $\frac{1}{4}$ C. $\frac{5}{32}$ D. $\frac{25}{150}$ E. $\frac{1}{25}$

6. If a person is randomly selected, what is the probability that the person will weigh more than 150 pounds?

A. $\frac{7}{32}$ B. $\frac{1}{2}$ C. $\frac{3}{8}$ D. $\frac{7}{16}$ E. $\frac{1}{4}$

7. If a person is randomly selected, what is the probability that the person will weigh under 100 pounds or over 200 pounds?

A. $\frac{3}{32}$ B. $\frac{1}{8}$ C. $\frac{3}{8}$ D. $\frac{7}{16}$ E. $\frac{7}{32}$

Questions 8 – 15 refer to Randal's chore chart for the month of May, shown below.

Sun	Mon	Tues	Wed	Thur	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

	Wash Dishes
	Take Out Trash
	Cook Dinner
	No Chore

8. If a day is selected at random from the month of May, what is the probability that it will be a trash day?

A. $\frac{4}{31}$ B. $\frac{5}{31}$ C. $\frac{1}{10}$ D. $\frac{3}{31}$ E. $\frac{7}{30}$

9. If a day is selected at random from the month of May, what is the probability that it will be a Saturday with no chores?

A. $\frac{3}{31}$ B. $\frac{3}{4}$ C. $\frac{1}{31}$ D. $\frac{1}{10}$ E. $\frac{1}{4}$

10. If a day is selected at random from the month of May, what is the probability that it will be a cook dinner day or a wash dishes day?

A. $\frac{11}{31}$ B. $\frac{13}{31}$ C. $\frac{12}{31}$ D. $\frac{2}{5}$ E. $\frac{14}{31}$

11. If a day is selected at random from the month of May, what is the probability that Randal will have a chore to do?

A. $\frac{11}{31}$ B. $\frac{13}{31}$ C. $\frac{15}{31}$ D. $\frac{2}{5}$ E. $\frac{14}{31}$

12. If a day is selected at random from the month of May, what is the probability that it will be an odd numbered day and a take out the trash day?

A. $\frac{3}{31}$ B. $\frac{2}{31}$ C. $\frac{2}{3}$ D. $\frac{3}{5}$ E. $\frac{1}{2}$

13. If a day is selected at random from the month of May, what is the probability that it will be an odd numbered day or a take out the trash day?

A. $\frac{17}{31}$ B. $\frac{16}{31}$ C. $\frac{18}{31}$ D. $\frac{15}{31}$ E. $\frac{1}{2}$

14. If it is a Thursday in May, what is the probability that Randal will have a chore to do?

A. $\frac{3}{31}$ B. $\frac{2}{31}$ C. $\frac{2}{3}$ D. $\frac{3}{5}$ E. $\frac{1}{2}$

15. If it is a Monday or a Tuesday in May, what is the probability that Randal will not have any chores to do?

A. $\frac{1}{2}$ B. $\frac{10}{31}$ C. $\frac{1}{5}$ D. $\frac{2}{5}$ E. $\frac{3}{5}$

2. Probability Expressed as a Decimal or a Percent

To express a probability as a decimal or percent, first calculate the probability as a fraction, as shown in the previous section.
Then, convert to decimal or percent as shown in Example One below.

How do you know if you need to convert to a decimal or a percent?
Look at the multiple choice answers. If they are decimals or percents, you must convert the fraction.

Example One
A bag of marbles has 20 red, 20 yellow, 10 white, 10 green, 20 black, and 10 blue.
What is the probability of randomly picking a red marble, expressed as a percent?

First, calculate the probability as a fraction.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{20 red marbles}}{\text{90 total marbless}} \rightarrow \frac{20}{90}$$

Second, convert the fraction to a decimal.
Think of the fraction bar as a division sign and divide. $\frac{20}{90} \rightarrow 20 \div 90 = 0.222$

Third, convert the decimal to a percent.
Move the decimal point 2 places to the right and add a percent sign. 0.222 = 22.2%
OR, multiply x 100 and add a percent sign. 0.222 x 100 = 22.2%

Note – You may have noticed that probability calculations are very similar to the percent calculations done in Lesson 3 of the Percent Word Problems section, where a fraction is formed with the part you want on top and the whole thing on the bottom.

Example Two
A bag of marbles has 20 red, 20 yellow, 10 white, 10 green, 20 black, and 10 blue. What is the probability of randomly picking a marble that is anything but white, expressed as a percent?

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{80 not white marbles}}{\text{90 total marbles}} \qquad \frac{80}{90} \rightarrow 80 \div 90 = 0.889 = 88.9\%$$

NOTE – If you know you have to convert to a decimal or a percent, there is no need to take the time to reduce the fraction before converting.

NOTE – How do you know how many decimal places to round to?

In Example Two, when you divide 80 ÷ 90, the calculator displays 0.88888888888 and the 8s continue forever. How do you know whether to round it to 0.9, 0.89, 0.889, etc.?

All are correct. Look at your multiple choice answers, and round it to the same number of places as you see there.

Practice Two *Answers – p. 21*

1. Convert each fraction to a decimal and then to a percent.

A. $\frac{3}{5}$ **B.** $\frac{5}{9}$

C. $\frac{1}{6}$ **D.** $\frac{4}{20}$

E. $\frac{7}{18}$ **F.** $\frac{6}{30}$

G. $\frac{3}{4}$ **H.** $\frac{15}{65}$

2. A case of 36 cans of peas was dropped and 6 of the cans were dented. What is the probability of randomly selecting a dented can?

A. 6% B. 15.3% C. 16.7% D. 60% E. 6.6%

3. In a 30 piece assortment of white and dark chocolate, 20 of the pieces are dark chocolate. What is the probability that a randomly selected piece of candy will be white?

A. 23% B. 20% C. 10% D. 33% E. 50%

4. A child's bag of blocks contains 12 red blocks, 15 green blocks, 10 blue blocks, and 14 white blocks. What is the probability that a randomly selected block will be green?
A. 0.29 B. 0.41 C. 0.51 D. 0.15 E. 0.25

Questions 5 – 8 refer to the following table of test scores for a group of students.

Score	Number of Students
90 – 100	16
80 – 89	24
70 – 79	40
60 – 69	35
Under 60	25

5. What is the probability that a randomly selected score will be in the 80 – 89 range?
A. 24% B. 17% C. 11% D. 35% E. 71%

6. What is the probability that a randomly selected score will be under 60?
A. 0.43 B. 0.6 C. 0.81 D. 0.25 E. 0.18

7. What is the probability that a randomly selected score will be over 69?
A. 29% B. 17% C. 57% D. 11% E. 80%

8. If a score is selected at random, into which of the score ranges below is it most likely to fall?
A. Over 89 B. Under 60 C. 80 – 100 D. Over 69 E. 60 – 79

3. Calculating Desired or Total Outcomes When Given Probability

A probability expressed as a fraction can be thought of as parts out of a whole.
Consider the following probability problem.

A bag of 64 candies has 40 fruit candies and 24 mint candies. What is the probability that a candy picked at random will be a fruit candy?

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{40 fruit candies}}{\text{64 total candies}} \rightarrow \frac{40}{64} = \frac{5}{8}$$

The probability $\frac{5}{8}$, expressed as a fraction, can be thought of as parts out of a whole.
If all the candy is divided into 8 equal parts, 5 of those parts are fruit candy.

That leaves 3 parts for the rest of the candy, and since there is only one other type of candy, those 3 parts are mint candies.
The fraction $\frac{3}{8}$ is the probability of picking a mint candy.

So, in a probability problem with just two possible outcomes, if you know the probability of one outcome, you can get the probability of the other outcome.
Use this concept in the following problems, which are different from the problems we have done so far.

All the problems we have done so far ask you to ***calculate a probability*** after being given total outcomes and a desired outcome.
In the following problems, you will be given a probability and asked to ***calculate the total or the desired outcomes***.

Example One

A bag is filled with cherry and orange candies. There are 36 cherry candies and the probability that a cherry candy will be picked at random from the bag is $\frac{3}{4}$. How many total candies are in the bag?

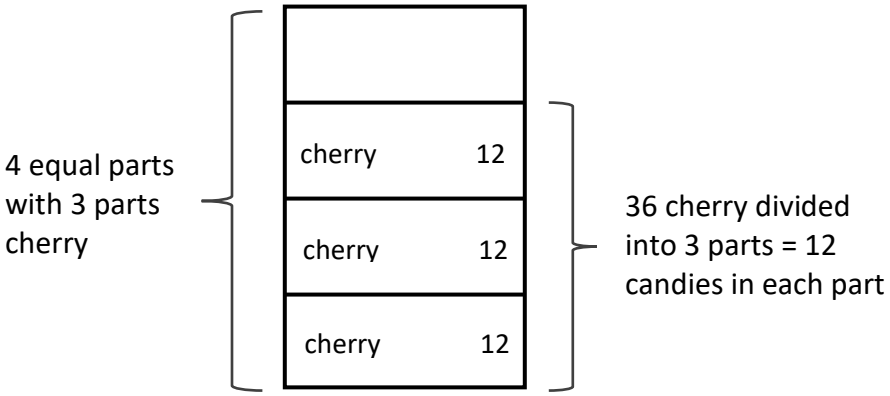
A. 60 B. 12 C. 48 D. 36 E. 24

Since the probability of getting cherry is $\frac{3}{4}$, think of the bag of candies as divided into 4 equal parts with 3 parts cherry.

The problem tells you there are 36 cherry candies, so divide the 36 cherry candies into 3 cherry parts to see how many candies are in 1 part. 36 ÷ 3 = 12
If 1 part is 12 candies, then the whole bag, or 4 parts, is 4 x 12 = 48 candies.
Answer: C. 48

You can draw a picture that shows all the information that is given in the problem.

Cherry probability of $\frac{3}{4}$ → 4 equal parts with 3 parts cherry.
36 cherry candies → divided into 3 cherry parts means 12 candies in each part.



Looking at the diagram of given information, you can see that the remaining part will be 12 orange candies, and the total number of candies is 4 x 12 = 48.

This type of problem can also be solved by setting up a proportion or an algebraic equation. If you are familiar with either of these methods, it may be a faster way for you to solve this kind of problem.

Set up a proportion.

$\frac{\text{cherry candies}}{\text{total candies}}$ $\frac{36}{x} = \frac{3}{4}$ $x = 4 \times 36 \div 3 =$ **48 total candies**

Set up an algebraic equation where x = total candies.

$$\frac{3}{4}x = 36$$
$$(\frac{4}{3})(\frac{3}{4}x) = (\frac{4}{3})(36)$$
$$x = \textbf{48 total candies}$$

Example Two

A jar is filled with black and white marbles. There are 30 white marbles and the probability of randomly picking a black marble is $\frac{5}{8}$. How many black marbles are in the jar?

A. 10 B. 50 C. 80 D. 30 E. 36

Since the probability of black is $\frac{5}{8}$, think of the jar as divided into 8 equal parts, with 5 parts black. If 5 of the 8 parts are black, then the remaining 3 parts must be white.

The problem tells you there are 30 white marbles, so divide the 30 white marbles into 3 white parts to see how many marbles are in 1 part. $30 \div 3 = 10$
The question asks for black marbles, which are 5 parts, or $5 \times 10 = 50$.
Answer: B. 50

OR – Set up a proportion. $\frac{\text{white marbles}}{\text{black marbles}}$ $\frac{30}{x} = \frac{3}{5}$ $x = 5 \times 30 \div 3 =$ **50 black marbles**

CAREFUL – The problem could have asked for total marbles or black marbles. Make sure to answer the question being asked, which is the number of black marbles, not the number of total marbles.

Example Three

A bag is filled with 84 pencils, some red and some black. If the probability of randomly picking a red pencil is $\frac{2}{7}$, how many black pencils are there?

A. 12 B. 24 C. 42 D. 36 E. 60

Since the probability of red is $\frac{2}{7}$, think of the bag as divided into 7 equal parts, with 2 parts red pencils. If 2 of the 7 parts are red, then the remaining 5 parts must be black.

The problem tells you there are 84 total pencils, and you know there are 7 total parts, so divide the 84 total pencils by 7 to see how many pencils are in 1 part. $84 \div 7 = 12$
The problem asks for black pencils, which are 5 parts, or $5 \times 12 = 60$.
Answer: E. 60

OR – Set up a proportion. $\frac{\text{black pencils}}{\text{total pencils}}$ $\frac{x}{84} = \frac{5}{7}$ $x = 5 \times 84 \div 7 =$ **60 black pencils**

OR – Because you are given the total, this can be solved with multiplication.
$\frac{5}{7}$ of the 84 pencils are black → $\frac{5}{7} \times 84 =$ **60 black pencils**

NOTE – In general, for this type of problem, the goal is to use the information given to see how many items are in 1 part, and then multiply times the number of parts that are in what the question is asking for.

Practice Three *Answers – p. 23*

1. A bag is filled with green and red cubes. The probability that a green cube will be picked at random from the bag is $\frac{2}{5}$. How many total cubes are in the bag if there are 10 green cubes?

A. 36 B. 12 C. 25 D. 48 E. 24

2. A pastry tray has two kinds of pastries, fruit and chocolate, and the probability that a fruit pastry will be picked at random is $\frac{1}{3}$. If there are 8 fruit pastries, how many chocolate pastries are there?

A. 32 B. 16 C. 8 D. 48 E. 24

3. A grab bag is filled with gold and silver rings. If there are 90 gold rings and the probability of randomly picking a silver ring is $\frac{4}{7}$, how many total rings are in the bag?

A. 200 B. 30 C. 120 D. 90 E. 210

4. A group of volunteers is picking slips at random out of a bag that will assign them to either a planning committee or a fundraising committee. If there are 8 planning slips and the probability of picking a fundraising slip is $\frac{3}{4}$, how many fundraising slips are in the bag?

A. 24 B. 8 C. 32 D. 16 E. 42

5. A class of students will be randomly assigned to either Group X or Group Y for a project. If there are 20 total students and the probability of being assigned to Group X is $\frac{2}{5}$, how many students will there be in group X?

A. 4 B. 12 C. 8 D. 20 E. 5

6. A jar with yellow and purple marbles has a total of 240 marbles. If the probability of randomly picking a yellow marble is $\frac{5}{12}$ how many purple marbles are in the jar?

A. 12 B. 100 C. 200 D. 140 E. 5

ANSWER KEY Lesson 1 Beginning Probability

Practice One

1. A book club has 20 women and 5 men as members. If one person is selected at random to choose the book for next month, what is the probability that it will be a man?

A. $\frac{1}{4}$ B. $\frac{4}{1}$ C. $\frac{5}{1}$ D. $\frac{1}{5}$ E. $\frac{4}{5}$

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{5 men}}{\text{25 total people}} \rightarrow \frac{5}{25} = \frac{1}{5}$

Answer: D. $\mathbf{\frac{1}{5}}$

NOTE – The total outcomes number is not given. You must add 20 women + 5 men to get the 25 total outcomes.

Questions 2 – 4 refer to a bag of marbles that contains 20 red marbles, 30 blue marbles, 14 white marbles, and 40 black marbles.

2. What is the probability that a randomly selected marble will be black?

A. $\frac{13}{5}$ B. $\frac{5}{13}$ C. $\frac{40}{1}$ D. $\frac{4}{9}$ E. $\frac{4}{5}$

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{40 black marbles}}{\text{104 total marbles}} \rightarrow \frac{40}{104} = \frac{5}{13}$

Answer: B. $\mathbf{\frac{5}{13}}$

3. What is the probability that a randomly selected marble will be white?

A. $\frac{7}{52}$ B. $\frac{7}{30}$ C. $\frac{7}{90}$ D. $\frac{14}{50}$ E. $\frac{14}{90}$

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{14 white marbles}}{\text{104 total marbles}} \rightarrow \frac{14}{104} = \frac{7}{52}$

Answer: A. $\mathbf{\frac{7}{52}}$

4. What is the probability that a randomly selected marble will be either red or blue?

A. $\frac{15}{26}$ B. $\frac{5}{26}$ C. $\frac{15}{52}$ D. $\frac{24}{52}$ E. $\frac{25}{52}$

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{50 marbles that are red or blue}}{\text{104 total marbles}} \rightarrow \frac{50}{104} = \frac{25}{52}$

Answer: E. $\mathbf{\frac{25}{52}}$

Questions 5 – 7 refer to the following table which shows the weights of a group of patients.

Weight in Pounds	Number of People
Under 100	15
100 – 125	25
126 – 150	40
151 – 175	35
176 – 200	25
Over 200	20

5. If a person is randomly selected what is the probability that the person will weigh 100 to 125 pounds?

A. $\frac{5}{28}$ B. $\frac{1}{4}$ C. $\frac{5}{32}$ D. $\frac{25}{150}$ E. $\frac{1}{25}$

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{25 people weigh 100 – 125 pounds}}{\text{160 total people}} \rightarrow \frac{25}{160} = \frac{5}{32}$$

Answer: C. $\mathbf{\frac{5}{32}}$

6. If a person is randomly selected, what is the probability that the person will weigh more than 150 pounds?

A. $\frac{7}{32}$ B. $\frac{1}{2}$ C. $\frac{3}{8}$ D. $\frac{7}{16}$ E. $\frac{1}{4}$

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{80 people over 150 pounds}}{\text{160 total people}} \rightarrow \frac{80}{160} = \frac{1}{2}$$

To get the number of desired outcomes, add together the number of people in every category that is more than 150 pounds. The 3 categories at the bottom of the table are all over 150 pounds, so add 35 + 25 + 20 = 80 to get desired outcomes.

Answer: B. $\mathbf{\frac{1}{2}}$

7. If a person is randomly selected, what is the probability that the person will weigh under 100 pounds or over 200 pounds?

A. $\frac{3}{32}$ B. $\frac{1}{8}$ C. $\frac{3}{8}$ D. $\frac{7}{16}$ E. $\frac{7}{32}$

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{35 people under 100 or over 200 pounds}}{\text{160 total people}} \rightarrow \frac{35}{160} = \frac{7}{32}$$

Answer: E. $\mathbf{\frac{7}{32}}$

Questions 8 – 15 refer to Randal's chore chart for the month of May, shown below.

Sun	Mon	Tues	Wed	Thur	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

	Wash Dishes
	Take Out Trash
	Cook Dinner
	No Chore

8. If a day is selected at random from the month of May, what is the probability that it will be a trash day?

A. $\frac{4}{31}$ B. $\frac{5}{31}$ C. $\frac{1}{10}$ D. $\frac{3}{31}$ E. $\frac{7}{30}$

To get desired outcomes, count up the dark gray squares.

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{3 trash days}}{\text{31 total days}} \rightarrow \frac{3}{31}$

Answer: D. $\mathbf{\frac{3}{31}}$

9. If a day is selected at random from the month of May, what is the probability that it will be a Saturday with no chores?

A. $\frac{3}{31}$ B. $\frac{3}{4}$ C. $\frac{1}{31}$ D. $\frac{1}{10}$ E. $\frac{1}{4}$

To be a desired outcome, a day must be both a Saturday and a day with no chores.

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{3 no chore Saturdays}}{\text{31 total days}} \rightarrow \frac{3}{31}$

Answer: A. $\mathbf{\frac{3}{31}}$

In question #9, the day is selected at random from the whole month.
Don't confuse this with a question that asks: If it is Saturday, what is the probability of a day with no chores? For that question, the day would be selected at random from just the Saturdays.

10. If a day is selected at random from the month of May, what is the probability that it will be a cook dinner day or a wash dishes day?

A. $\frac{11}{31}$ B. $\frac{13}{31}$ C. $\frac{12}{31}$ D. $\frac{2}{5}$ E. $\frac{14}{31}$

To get desired outcomes, count up all the days that are either light gray or black.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{12 days are cook dinner or wash dishes}}{\text{31 total days}} \rightarrow \frac{12}{31}$$

Answer: C. $\mathbf{\frac{12}{31}}$

11. If a day is selected at random from the month of May, what is the probability that Randal will have a chore to do?

A. $\frac{11}{31}$ B. $\frac{13}{31}$ C. $\frac{15}{31}$ D. $\frac{2}{5}$ E. $\frac{14}{31}$

To get desired outcomes, count up all the days with any chore, which will be the black squares, the dark gray squares, and the light gray squares.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{15 days have a chore}}{\text{31 total days}} \rightarrow \frac{15}{31}$$

Answer: C. $\mathbf{\frac{15}{31}}$

12. If a day is selected at random from the month of May, what is the probability that it will be an odd numbered day and a take out the trash day?

A. $\frac{3}{31}$ B. $\frac{2}{31}$ C. $\frac{2}{3}$ D. $\frac{3}{5}$ E. $\frac{1}{2}$

The use of the word "and" means a desired outcome must meet both requirements. Desired outcomes must be ***both*** odd numbered ***and*** colored dark gray. Days 3 and 17 are the only days that meet both requirements.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{2 days are both odd and trash}}{\text{31 total days}} \rightarrow \frac{2}{31}$$

Answer: B. $\mathbf{\frac{2}{31}}$

13. If a day is selected at random from the month of May, what is the probability that it will be an odd numbered day or a take out the trash day?

A. $\frac{17}{31}$ B. $\frac{16}{31}$ C. $\frac{18}{31}$ D. $\frac{15}{31}$ E. $\frac{1}{2}$

The use of the word "or" means a desired outcome must meet one of the requirements, but doesn't have to meet both. Desired outcomes are all the days that are ***either*** odd ***or*** colored dark gray for a take out the trash day. If it meets either requirement, count it as a desired outcome.

Days 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 24, 25, 27, 29, and 31 meet one of the requirements.

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{17 days are either odd or trash}}{\text{31 total days}} \rightarrow \frac{17}{31}$$

Answer: A. $\mathbf{\frac{17}{31}}$

Be careful not to count up all the odd days and then all the dark gray take out the trash days. That would give you 16 odd plus 3 trash for a total of 19 desired outcomes. This is incorrect because it double counts the days that are both odd and a take out the trash day.

14. If it is a Thursday in May, what is the probability that Randal will have a chore to do?

A. $\frac{3}{31}$ B. $\frac{2}{31}$ C. $\frac{2}{3}$ D. $\frac{3}{5}$ E. $\frac{1}{2}$

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{2 Thursdays with a chore}}{\text{4 total Thursdays}} \rightarrow \frac{2}{4} = \frac{1}{2}$$

Answer: E. $\mathbf{\frac{1}{2}}$

NOTE – You are told it is a Thursday, so there are only 4 possible outcomes since there are only 4 Thursdays. The day is not selected at random from the whole month. It is selected at random from all the Thursdays.

15. If it is a Monday or a Tuesday in May, what is the probability that Randal will not have any chores to do?

A. $\frac{1}{2}$ B. $\frac{10}{31}$ C. $\frac{1}{5}$ D. $\frac{2}{5}$ E. $\frac{3}{5}$

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{4 no chore days on Monday or Tuesday}}{\text{10 total Mondays or Tuesdays}} \rightarrow \frac{4}{10} = \frac{2}{5}$$

Answer: D. $\mathbf{\frac{2}{5}}$

NOTE – You are told it is Monday or Tuesday, so there are only 10 possible outcomes since there are only 10 days that are Monday or Tuesday. The day is not selected at random from the whole month. It is selected at random from all the Mondays and Tuesdays.

Practice Two

1. Convert each fraction to a decimal and then to a percent.

A. $\frac{3}{5} = 3 \div 5 =$ **0.6 = 60%**

B. $\frac{5}{9} = 5 \div 9 =$ **0.556 = 55.6%**

C. $\frac{1}{6} = 1 \div 6 =$ **0.167 = 16.7%**

D. $\frac{4}{20} = 4 \div 20 =$ **0.2 = 20%**

E. $\frac{7}{18} = 7 \div 18 =$ **0.389 = 38.9%**

F. $\frac{6}{30} = 6 \div 30 =$ **0.2 = 20%**

G. $\frac{3}{4} = 3 \div 4 =$ **0.75 = 75%**

H. $\frac{15}{65} = 15 \div 65 =$ **0.231 = 23.1%**

2. A case of 36 cans of peas was dropped and 6 of the cans were dented. What is the probability of randomly selecting a dented can?

A. 6% B. 15.3% C. 16.7% D. 60% E. 6.6%

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{6 dented cans}}{\text{36 total cans}} \rightarrow \frac{6}{36} = 6 \div 36 = 0.167 = 16.7\%$

Answer: C. 16.7%

3. In a 30 piece assortment of white and dark chocolate, 20 of the pieces are dark chocolate. What is the probability that a randomly selected piece of candy will be white?

A. 23% B. 20% C. 10% D. 33% E. 50%

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{10 white pieces}}{\text{30 total pieces}} \rightarrow \frac{10}{30} = 10 \div 30 = 0.33 = 33\%$

Answer: D. 33%

4. A child's bag of blocks contains 12 red blocks, 15 green blocks, 10 blue blocks, and 14 white blocks. What is the probability that a randomly selected block will be green?

A. 0.29 B. 0.41 C. 0.51 D. 0.15 E. 0.25

$\frac{\text{desired outcomes}}{\text{total possible outcomes}}$ $\frac{\text{15 green blocks}}{\text{51 total blocks}} \rightarrow \frac{15}{51} = 15 \div 51 = 0.29$

Answer: A. 0.29

Questions 5 – 8 refer to the following table of test scores for a group of students.

Score	Number of Students
90 – 100	16
80 – 89	24
70 – 79	40
60 – 69	35
Under 60	25

5. What is the probability that a randomly selected score will be in the 80 – 89 range?
A. 24% B. 17% C. 11% D. 35% E. 71%

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{24 students with 80–89 scores}}{\text{140 total students}} \rightarrow \frac{24}{140} = 24 \div 140 = 0.17 = 17\%$$

Answer: B. 17%

6. What is the probability that a randomly selected score will be under 60?
A. 0.43 B. 0.6 C. 0.81 D. 0.25 E. 0.18

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{25 students with under 60 scores}}{\text{140 total students}} \rightarrow \frac{25}{140} = 25 \div 140 = 0.18$$

Answer: E. 0.18

7. What is the probability that a randomly selected score will be over 69?
A. 29% B. 17% C. 57% D. 11% E. 80%

$$\frac{\text{desired outcomes}}{\text{total possible outcomes}} \qquad \frac{\text{80 students with over 69 scores}}{\text{140 total students}} \rightarrow \frac{80}{140} = 80 \div 140 = 0.57 = 57\%$$

To get desired outcomes, add up the students in all the ranges that meet the requirement of being over 69. The top 3 ranges on the chart all meet the requirement, so add up 16 + 24 + 40 = 80.
Answer: C. 57%

8. If a score is selected at random, into which of the score ranges below is it most likely to fall?
A. Over 89 B. Under 60 C. 80 – 100 D. Over 69 E. 60 – 79

The fastest way to do this is to realize that the range with the most students will have the highest probability of being selected. Calculate the size of each score range and select the one with the most students.
A. Over 89 16 students
B. Under 60 25 students
C. 80 – 100 40 students (24 + 16 = 40)
D. Over 69 80 students (40 + 24 + 16 = 80)
E. 60 – 79 75 students (35 + 40 = 75)
Answer: D. Over 69 *(Explanation continued below)*

You could also calculate the probability of a score falling in each of the ranges and then select the highest probability, but this will take longer.
Look at the probability calculations below. They all have the same number on the bottom of the fraction, so the calculation with the biggest number on top of the fraction, which is the biggest range, will have the highest probability.

A. Over 89 16 students $\frac{16}{140} = 16 \div 140 = 0.114 = 11.4\%$

B. Under 60 25 students $\frac{25}{140} = 25 \div 140 = 0.179 = 17.9\%$

C. 80 – 100 40 students $\frac{40}{140} = 40 \div 140 = 0.286 = 28.6\%$

D. Over 69 80 students $\frac{80}{140} = 80 \div 140 = 0.571 =$ **57.1%**

E. 60 – 79 75 students $\frac{75}{140} = 75 \div 140 = 0.536 = 53.6\%$

Practice Three

1. A bag is filled with green and red cubes. The probability that a green cube will be picked at random from the bag is $\frac{2}{5}$. How many total cubes are in the bag if there are 10 green cubes?

A. 36 B. 12 C. 25 D. 48 E. 24

Since the probability of getting green is $\frac{2}{5}$, think of the bag of cubes as divided into 5 equal parts with 2 parts green.

The problem tells you there are 10 green cubes, so divide the 10 green cubes into 2 green parts to see how many cubes are in 1 part. $10 \div 2 = 5$
If 1 part is 5 cubes, then the whole bag, or 5 parts, is $5 \times 5 = 25$ cubes.
Answer: C. 25

OR – Set up a proportion. $\frac{\text{green cubes}}{\text{total cubes}}$ $\frac{10}{x} = \frac{2}{5}$ $x = 5 \times 10 \div 2 =$ **25 total cubes**

2. A pastry tray has two kinds of pastries, fruit and chocolate, and the probability that a fruit pastry will be picked at random is $\frac{1}{3}$. If there are 8 fruit pastries, how many chocolate pastries are there?

A. 32 B. 16 C. 8 D. 48 E. 24

Since the probability of getting fruit is $\frac{1}{3}$, think of the tray of pastries as divided into 3 equal parts with 1 part fruit. The problem tells you there are 8 fruit pastries, so there are 8 pastries in 1 part.

If 1 part is fruit, the remaining 2 parts must be chocolate.
The question asks for chocolate pastries, which are 2 parts, or 2 x 8 = 16.
Answer: B. 16

OR – The problem tells you that fruit = 8 = $\frac{1}{3}$ of the total. If $\frac{1}{3}$ of the total is 8, then the remaining $\frac{2}{3}$, which are chocolate, must be twice as big. 2 x 8 = **16 chocolate pastries**

OR – Set up a proportion. $\frac{\text{fruit pastries}}{\text{choc. pastries}}$ $\frac{8}{x} = \frac{1}{2}$ x = 2 x 8 ÷ 1 = **16 chocolate pastries**

3. A grab bag is filled with gold and silver rings. If there are 90 gold rings and the probability of randomly picking a silver ring is $\frac{4}{7}$, how many total rings are in the bag?

A. 200 B. 30 C. 120 D. 90 E. 210

Since the probability of silver is $\frac{4}{7}$, think of the bag of rings as divided into 7 equal parts with 4 parts silver. If 4 parts are silver, then the remaining 3 parts must be gold.

The problem tells you there are 90 gold rings, so divide the 90 gold rings into 3 gold parts to see how many rings are in 1 part. 90 ÷ 3 = 30
If 1 part is 30 rings, then the whole bag, or 7 parts, is 7 x 30 = 210 rings.
Answer: E. 210

OR – Set up a proportion. $\frac{\text{gold rings}}{\text{total rings}}$ $\frac{90}{x} = \frac{3}{7}$ x = 7 x 90 ÷ 3 = **210 total rings**

4. A group of volunteers is picking slips at random out of a bag that will assign them to either a planning committee or a fundraising committee. If there are 8 planning slips and the probability of picking a fundraising slip is $\frac{3}{4}$, how many fundraising slips are in the bag?

A. 24 B. 8 C. 32 D. 16 E. 42

Since the probability of fundraising is $\frac{3}{4}$, think of the slips as divided into 4 equal parts, with 3 parts fundraising slips. If 3 of the 4 parts are fundraising, then the remaining 1 part must be planning.

The problem tells you there are 8 planning slips, so there are 8 slips in 1 part.
The question asks for fundraising slips, which are 3 parts. 3 x 8 = 24.
Answer: A. 24

OR – Set up a proportion. $\frac{\text{plan. slips}}{\text{fund. slips}}$ $\frac{8}{x} = \frac{1}{3}$ $x = 3 \times 8 \div 1 =$ **24 fundraising slips**

5. A class of students will be randomly assigned to either Group X or Group Y for a project. If there are 20 total students and the probability of being assigned to Group X is $\frac{2}{5}$, how many students will there be in group X?

A. 4 B. 12 C. 8 D. 20 E. 5

Since the probability of Group X is $\frac{2}{5}$, think of all the students as divided into 5 equal parts with 2 parts Group X.

The problem tells you there are 20 total students, and you know there are 5 total parts, so divide the 20 total students by 5 to see how many students are in 1 part. $20 \div 5 = 4$
If 1 part is 4 students, then Group X, or 2 parts is 2 x 4 = 8 students.
Answer: C. 8

OR – Because you are given the total, this can be solved with multiplication.
$\frac{2}{5}$ of the 20 students are Group X → $\frac{2}{5}$ x 20 = **8 Group X students**

OR – Set up a proportion. $\frac{\text{Grp X students}}{\text{total students}}$ $\frac{x}{20} = \frac{2}{5}$ $x = 2 \times 20 \div 5 =$ **8 Grp X students**

6. A jar with yellow and purple marbles has a total of 240 marbles. If the probability of randomly picking a yellow marble is $\frac{5}{12}$ how many purple marbles are in the jar?

A. 12 B. 100 C. 200 D. 140 E. 5

Since the probability of yellow is $\frac{5}{12}$, think of the jar of marbles as divided into 12 equal parts with 5 parts yellow. If 5 parts are yellow, then the remaining 7 parts must be purple.

The problem tells you there are 240 total marbles, and you know there are 12 total parts, so divide the 240 total marbles by 12 to see how many marbles are in 1 part.
240 ÷ 12 = 20
If 1 part is 20 marbles, then purple marbles, or 7 parts is 7 x 20 = 140 marbles.
Answer: D. 140

OR – Because you are given the total, this can be solved with multiplication.
$\frac{7}{12}$ of the 240 marbles are purple → $\frac{7}{12}$ x 240 = **140 purple marbles**

OR – Set up a proportion. $\frac{\text{purple marbles}}{\text{total marbles}}$ $\frac{x}{240} = \frac{7}{12}$ x = 7 x 240 ÷ 12 = **140 purple**